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The Locating Chromatic Number for Pizza Graphs

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ABSTRACT

The location chromatic number for a graph is an extension of the concepts of partition dimension and vertex coloring in a graph. The minimum number of colors required to perform location coloring in graph G is referred to as the location chromatic number of graph G. This research is a literature study that discusses the location chromatic number of the Pizza graph. The approach used to calculate the location-chromatic number of these graphs involves determining upper and lower bounds. The results obtained show that the location chromatic number of the pizza graph is 4 for $n = 3$ and n for $n \geq 4$.

Keywords: Location Chromatic Number, Pizza Graph, Color Code

INTRODUCTION

In this increasingly advanced era, mathematics has become an important tool for solving complex problems using modeling methods. Mathematical modeling allows us to describe and analyze real-life situations, such as mapping areas, determining the shortest routes, and exploring other phenomena. Graph theory is one of the most important and evolving branches of mathematics (Surbakti & Ramadhani, 2022).

Graphs are used to represent a variety of real-world situations, such as social networks (Bhatti et al., 2023). Research in the field of graph theory plays an important role in a wide range of disciplines, including computer science, optimization, logistics, and networking.

One of the interesting topics in graph theory is location coloring on a graph, which is an extension of the node coloring and partition dimensions on the graph.

Location coloring in a graph is an attempt to determine the minimum number of colors needed to color all the nodes on the graph, provided that no two nodes are connected by sides that have the same color (Asmiati et al., 2018). Node coloring has various of benefits in everyday life, such as scheduling, radio frequency determination, and games (Surbakti, 2023).

Node coloring on the graph $G = (V, E)$ refers to the process of mapping each node v into the set of native numbers $c(v)$, where $c: V \rightarrow N$, in such a way that each adjacent node has a different color (i.e., $c(v) \neq c(w)$). If the number of colors used is k, then G is said to have k-coloring. The chromatic number of G, marked with $\chi(G)$, is the smallest native number of k that allows coloring k where each of the two adjacent nodes has a different color. The chromatic number of the location of G is marked by $\chi L(G)$ and is the least native number of k, which allows the coloring of a location with k different colors for G. In this context, $\chi\,(G) \leq \chi L(G)$ for each graph connected to G, since each coloring location is also a coloring (Harary & Melter, 1976).

Definition 1.1 (Chartrand et al., 2002) Let G be a finite and connected graph. Let c be the appropriate coloring of a connected graph G, where m is a positive integer, and the colors used are $1, 2, \cdots, and\ m$. Thus, the coloring c can be regarded as a partition Π of $V(G)$ into color classes (independent sets) $C_1, C_2, \cdots, C_m$, where the nodes of C_j are colored by j for $1 \leq j \leq m$. The *color code* $c_\Pi(u)$ of a node u in G is the ordered m −tuple $\left(d(u, C_1), \cdots, (du, C_m)\right)$ where $d\left(u, C_j\right) = \min\{d(u, x) | x \in C_j\}$ for $1 \leq j \leq m$. If all vertices in $V(G)$ have distinct color codes, then the coloring is desired to locate coloring. A minimum locating-coloring uses a minimum number of colors and this number is called the locating-chromatic number of graph G, marked by $\chi L(G)$.

Chartrand and his colleagues have determined the chromatic number of locations for some types of graphs, such as tracks, cycles, complete multipartite graphs, and double-star graphs (Chartrand et al., 2002).

Research into the chromatic number of locations in the context of graphs remains interesting to this day, as no theorem can provide a definitive solution to the calculation of the chromatic number of places on all types of graphs. In 2021, Irawan et al. determined the chromatic number of locations on the origami graph. Then Rahmatalia, *et al.* (2022) determine the chromatic number of the locations of the trajectory split graph (Rahmatalia et al., 2022). Then, in 2023, Wellyanti *et al.* defined the chromatic number of locations in the lobster graph (Welyyanti et al., 2023). In the same year, there were several studies related to the location of chromatic numbers for Certain Operations of Origami Graphs (Asmiati et al., 2023) and edge amalgamation graphs of star graphs with order $m + 1$ and complete graphs with order n (Hartiansyah & Darmaji, 2023).

From the description above, researchers are interested in exploring the chromatic number of locations on the pizza graph. The pizza graph noted by Pz_n is a graph with $V(Pz_n) = \{u, v_i, w_i: 1 \leq i \leq n\}$ and $E(Pz_n) = \{uv_i, v_iw_i: 1 \leq i \leq n\} \cup \{w_iw_{i+1}: 1 \leq i \leq n - 1\} \cup \{w_nw_1\}$. The pizza graph $Pz_n$is a graph with $2n + 1$ nodes obtained from a subdivision of the W_n-wheel graph on each finger (Nabila & Salman, 2015).

Theorem 1.1 (Behtoei, 2011) For $n \geq 3$, Let $W_n = K_1 + C_n$ and $l = \min\{k \in N | \, n \leq \frac{1}{2}(k^3 - k^2)\}$.

$$\chi_L(W_n) = \begin{cases} 1 + \chi_L(C_n), & if\ 3 \leq n < 9; \\ l + 1, if\ n \neq \dfrac{1}{2}(l^3 - l^2 - 1)\ and\ n \geq 9; \\ l + 2, if\ n = \dfrac{1}{2}(l^3 - l^2 - 1)\ and\ n \geq 9. \end{cases}$$

MATERIAL AND METHOD

The research is a literary study to determine the chromatic number of locations on the pizza graph. The research begins with defining the problem to be discussed. Step two: describe the pizza chart marked with Pz_n. The set of nodes and arches of Pz_n is defined as follows.

Definition 2.1 (Nabila & Salman, 2015) A pizza graph Pz_n is a graph with $V(Pz_n) = \{u, v_i, w_i: 1 \leq i \leq n\}$ and $E(Pz_n) = \{uv_i, v_iw_i: 1 \leq i \leq n\} \cup \{w_iw_{i+1}: 1 \leq i \leq n - 1\} \cup \{w_nw_1\}$.

As an illustration, Figure 1 is a pizza graph with 4 nodes marked with Pz_4.

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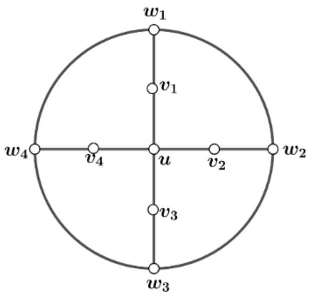

Figure 1. Pizza graph Pz_4

Step three: create a coloring c and a partition Π on $V(Pz_n)$, and proceed to determine $\chi_L(Pz_n)$. Step four, prove the chromatic number of locations on the pizza graph Pz_n. If $\chi_L(Pz_n) = 4$, $\chi_L(Pz_n) \geq 4$ and $\chi_L(Pz_n) \leq 4$ for $n = 3$. To prove the lower boundary of $\chi_L(Pz_n) \geq 4$, then refer to the **Theorem 1.1**. proving that the upper boundaries of $\chi_L(Pz_n) \leq 4$ by constructing the functions and color codes of $V(Pz_n)$. To prove the bottom limit of $V\chi_L(Pz_n) \geq n$, it will be shown that $n - 1$ color is not sufficient. Next, prove that $\chi_L(Pz_n) \leq n$ by constructing functions and color codes of $V(Pz_n)$. Step five, make a conclusion based on the analysis of the proven theorems.

RESULT AND DISCUSSION

Suppose $n \in$ N with $n \geq 3$ and W_n is a wheel graph with $n + 1$ nodes, which has a node in the center and adjacent to the entire nodes. A pizza graph with $2n + 1$ nodes, noted with Pz_n is a graph of a subdivision of the W_n wheel graph on each finger. A pizza graph with $Pz_n$is the graph with $V(Pz_n) = \{u, v_i, w_i: 1 \leq i \leq n\}\}$ and $E(Pz_n) = \{uv_i, v_iw_i: 1 \leq i \leq n\} \cup \{w_iw_{i+1}: 1 \leq i \leq n - 1\} \cup \{w_nw_1\}$.

Pizza Graph Location Chromatic Number

Theorem 3.1. Suppose n is a positive integer, for $n \geq 3$. The chromatic number of the location of the pizza graph Pz_n is vertical.

$$\chi_L(Pz_n) = \begin{cases} 4, & \text{for } n = 3; \\ n, & \text{for } n \geq 4. \end{cases}$$

The evidence is divided into two cases.

Case 1. For $n = 3$

First, it will prove the bottom limit of the chromatic number of locations of the pizza graph for $n = 3$. Since the pizza chart has a wheel chart with each bow adjacent to the central node, based on **Theorem 1.1** it is proved that $\chi_L(Pz_n) \geq 4$ for $n = 3$. Next, it will be shown that 4 is the upper boundary of the chromatic number of locations of the graph Pz_n. To prove the top boundaries, we only need to determine the presence of the optimum coloring of the location $c: V(Pz_n) \rightarrow \{1,2,3,4\}$. For $n = 3$, construct the function as follows:

$$c(u) = 4.$$
$$c(v_i) = \begin{cases} i + 1, for\ i \in [1,2]; \\ \quad 1, for\ i = n. \end{cases}$$
$$c(w_i\} = i, for\ i \in [1, n].$$

By using the coloring c, we obtain the color codes of $V(Pz_n)$ as follows:

$$c_\pi(u) = \begin{cases} 0, for\ 4^{th}\ component; \\ 1, otherwise. \end{cases}$$

$$c_\pi(v_i) = \begin{cases} 0, for\ i + 1^{th}\ component, i \in [1,2]; \\ 0, for\ 1^{st}\ component, i = n; \\ 1, for\ i^{th}\ component, i \in [1, n]; \\ 1, for\ 4^{th}\ component, i \in [1, n]; \\ 2, otherwise. \end{cases}$$

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$$c_\pi(w_i) = \begin{cases} 0, for\ i^{th}\ component, i \in [1,n]; \\ 1, for\ i+1^{th}\ component, i \in [1,2]; \\ 1, for\ i-1^{th}\ component, i \in [2,n]; \\ 1, for\ n^{th}\ component, i = 1; \\ 1, for\ 1^{st}\ component, i = n; \\ 2, otherwise. \end{cases}$$

Based on **Definition 1.1** Since all vertices in $V(Pz_n)$ have distinct color codes, the coloring is desired to locate coloring. Thus, have distinct color codes, then the coloring is desired locating coloring. Thus, $\chi_L(Pz_n) = 4$.

Case 2. For $n \geq 4$

To determine the lower bound, we will show that $n-1$ colors are insufficient. For a contradiction, assume that there exists a $(n-1)$-locating coloring c on Pz_n for $n \geq 4$. We assign $\{c(u), c(v_i), c(w_i)\} = \{1,2,\cdots,n-1\}$. We have $c(u) \neq c(v_i)$ and $c(v_i) \neq c(w_i)$ because the vertices are adjacent. Let

$$c(u) = n-1.$$
$$c(v_i) = \begin{cases} i+1, for\ i \in [1,n-3]; \\ 1, for\ i \in [n-2,n]. \end{cases}$$
$$c(w_i\} = i, for\ i \in [1,n-1].$$

and otherwise are colored with 2. Thus $c_\pi(u) = c_\pi(w_{n-1})$ which is a contradiction. Thus, we have $\chi_L(Pz_n) \geq n$.

To show that n is an upper bound for the locating chromatic number of the pizza graph Pz_n, it suffices to prove the existence of an optimal locating coloring $c: V(Pz_n) \rightarrow \{1,2,\cdots,n\}$. For $n \geq 4$, we construct the function in the following way:

$$c(u) = n.$$
$$c(v_i) = \begin{cases} i+1, for\ i \in [1,n-2]; \\ 1, for\ i \in [n-1,n]. \end{cases}$$
$$c(w_i\} = i, for\ i \in [1,n].$$

By using the coloring c, we obtain the color codes of $V(Pz_n)$ as follows:

$$c_\pi(u) = \begin{cases} 0, for\ n^{th}\ component; \\ 1, otherwise. \end{cases}$$

$$c_\pi(v_i) = \begin{cases} 0, for\ i+1^{th}\ component, i \in [1,n-2]; \\ 0, for\ 1^{st}\ component, i \in [n-1,n]; \\ 1, for\ i^{th}\ component, i \in [1,n]; \\ 1, for\ 4^{th}\ component, \quad i \in [1,n-1]; \\ 2, otherwise. \end{cases}$$

$$c_\pi(w_i) = \begin{cases} 0, for\ i^{th}\ component, i \in [1,n]; \\ 1, for\ i+1^{th}\ component, i \in [1,n-1]; \\ 1, for\ i-1^{th}\ component, i \in [2,n]; \\ 1, for\ n^{th}\ component, i = 1; \\ 2, for\ i+2^{th}\ component, i \in [1,n-2]; \\ 2, for\ i-2^{th}\ component, i \in [n-1,n], n \geq 5; \\ 2, for\ n-1^{th}\ component, i = 1; \\ 2, for\ n^{th}\ component, i \in [2,n-2]; \\ 2, for\ 2^{nd}\ component, i = n; \\ 3, otherwise. \end{cases}$$

Since all vertices in $V(Pz_n)$ have distinct color codes, then the coloring is desired to locate coloring. Thus, have distinct color codes, then the coloring is desired locating coloring. Thus, $\chi_L(Pz_n) = n$.

Figure 2 shows the minimum location coloring Pz_4.

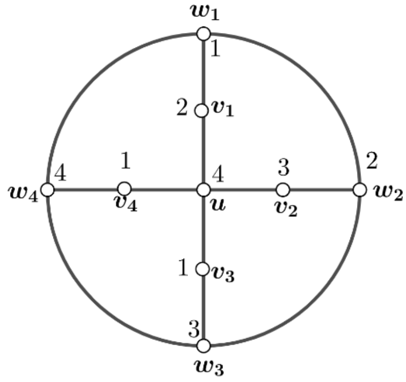

Figure 2. Minimum location coloring Pz_4

CONCLUSION

The Pizza graph's discovered locating chromatic number is

$$\chi_L(Pz_n) = \begin{cases} 4, & for\ n = 3; \\ n, & for\ n \geq 4. \end{cases}$$

The author suggests further research to be able to determine the chromatic number of locations from the specific graph results of the operation. For example, the graph comb operation results from a star graph and a cycle graph.

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