

Parameter Estimation of Logistic Growth Model for Covid-19 Cases in Lampung Using Particle Swarm Optimization

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ABSTRACT

Particle Swarm Optimization (PSO) is an optimization algorithm inspired by the behavior and movements of flocks of animals such as birds, fish, insects. In this study, we implement PSO algorithm to estimate the parameters of a mathematical model depicting population growth in the form logistics curve. The model is fitted to COVID-19 cumulative cases in Lampung Province, Indonesia. Based on the results obtained, PSO shows very good performance in estimating the parameters of the Covid-19 growth curve in Lampung Province, with a Mean Absolute Percentage Error (MAPE) value of all observations of less than 10%. We found that the MAPE decline as the number of particles increases.

Keywords: particle swarm optimization, parameter estimation, covid-19, data-driven model

INTRODUCTION

Harnessing the power of Particle Swarm Optimization (PSO), this research discusses a data-driven model for COVID-19 to understand its dynamics. While optimization serves as the underlying mechanism, the focal point rests on constructing an effective model to comprehend the evolution of COVID-19 cases. The PSO method, originally proposed by Kennedy and Eberhart in 1995, draws inspiration from natural collective behaviors seen in animal flocks, rendering it a prominent solution for addressing optimization challenges in diverse domains. This method is widely employed in applications like weather forecasting, scheduling, and result optimization (Muhamad et al., 2017; Rahmalia & Rahmalia, 2017; Dista & Abdulloh, 2022). Its lies in its simplicity, efficient implementation, and require low computational complexity.

In epidemiological research, the modeling of COVID-19 has become

crucial, particularly with regard to estimating cumulative data. Various modeling techniques have been explored in the recent past to gain insights into the dynamics of the pandemic. One prevalent approach is the use of autoregressive models, exemplified by the ARIMA (AutoRegressive Integrated Moving Average) framework Louisa et al. (2022). Researchers such as have investigated the application of ARIMA models to forecast COVID-19 trends. Additionally, in (Agosto and Giudici, 2020) and (Alabdulrazzaq et al., 2021) the ARIMA models is employed to analyze and predict COVID-19 data patterns.

Another exploration involves the application of Deep Learning Recurrent Neural Networks (RNNs) to model COVID-19 data, treating it as sequential information (Ghozi et al., 2022). The study finds RNNs to capture the inherent sequential dependencies within the pandemic data, enabling them to make

accurate predictions about its progression.

Within the domain of stochastic modeling, the uncertainty inherent in COVID-19 outbreaks can be delved by utilizing Brownian motion-based stochastic models (Edriani, 2021). The application the concept of geometric Brownian motion to introduce a novel perspective on the spread of COVID-19 has been studied in (Fabiano and Radenovic, 2021).

Compartmental models have also played a pivotal role in understanding dynamics of infectious diseases within a population (Rizka, 2022). In (Soewono, 2020), the model is employed to describe and analyze the spread of Covid-19 in Indonesia. However, since the model described by systems of differential equations, often necessitate numerical solutions, which are typically obtained through methods like the Runge-Kutta method or the Predictor-Corrector method, as demonstrated by (Fauzi and Wiryanto, 2018).

An alternative approach to modeling COVID-19 cumulative data that has gained attention is the utilization of logistics models (Nuraini et al., 2020), (Puspita et al., 2023). This approach is particularly appealing due to its relative simplicity, as the logistics model is already defined as a function with parameters, making the estimation process more straightforward and efficient. These diverse modeling techniques contribute to a comprehensive understanding of COVID-19 dynamics, offering valuable insights to inform public health strategies and decision-making processes.

This study intends to explore a crucial research problem within epidemiology, focusing on refining parameter estimation methods for COVID-19 modeling. Specifically, the research centers on utilizing the logistic growth curve to model the cumulative

cases of the pandemic. To enhance the precision and efficiency of parameter estimation, the study incorporates the influence of varying iteration number. This research aims to identify optimal parameter values that align most accurately with the data, with an additional dimension of evaluating the role of iteration in convergence. Furthermore, the investigation examines the impact of random initial values on the efficacy of the PSO algorithm in approaching the true parameters of the logistic growth curve model. This research has the potential to offer valuable insights into refining parameter estimation techniques for COVID-19 modeling, thereby aiding in a more comprehensive understanding and projection of the dynamics.

MATERIAL AND METHOD

Particle Swarm Optimization

$$v_k^{(i)} = wv_{k-1}^{(i)} + c_1r_1 \left[p_{best}^{(i)} - \vec{x}_{k-1}^{(i)} \right] + c_2r_2 \left[G_{best} - \vec{x}_{k-1}^{(i)} \right] \quad (1)$$

$$\vec{x}_k^{(i)} = v_k^{(i)} + \vec{x}_{k-1}^{(i)} \quad (2)$$

for $k = 0, 1, 2, \dots, k_{max}$ and

$i = 1, 2, \dots, N$, where

$v_k^{(i)}$: The velocity of particle i at iteration k

$\vec{x}_k^{(i)}$: The position of particle i at iteration k

and hyper-parameters

w : inertial weight

c_1, c_2 : acceleration coefficient

r_1, r_2 : random number (0,1)

N : number of particles

The beauty of nature which inspire this algorithm lies in the incorporation of the track record of its individual particle and recognizing its optimal value, namely

$$\vec{P}_{best}^{(i)} = \min_{x^{(i)}} \{f(x_1^{(i)}), f(x_2^{(i)}), \dots, f(x_k^{(i)})\} \quad (3)$$

Moreover, this algorithm also appreciate of the swarm best value, namely

$$\vec{G}_{best} = \min \{\vec{P}_{best}^{(1)}, \vec{P}_{best}^{(2)}, \dots, \vec{P}_{best}^{(N)}\} \quad (4)$$

In this study, the inertial weight is adjustable based on the iteration number following

$$w(k) = \left(\frac{k_{max}-k}{k_{max}}\right)(w_{max} - w_{min}) + w_{min} \quad (5)$$

where k_{max} is iteration number, just in case the hard to find its convergence criterion, w_{max} and w_{min} denoting given maximum and minimum allowed weight respectively.

Logistic Growth Model

The logistic function can be written as follows

$$Y_{\vec{\theta}}(t) = y_0 + \frac{y_{max}-y_0}{1+Ce^{-D(t-E)}} \quad (6)$$

The vector $\vec{\theta} = (y_0, y_{max}, C, D, E)$ will be the fitting parameters. The number y_0 and y_{max} denoting minimum and maximum number of cases respectively, and C is intrinsic growth constant. The logistic function has inflection point at $x = \frac{\ln C}{D} + E$ where the growth of the curves tend to slow down.

Objective Function

The optimization problem of this study will be minimizing the following unconstrained problem

$$F(\vec{\theta}) = \frac{1}{N} \sum_{i=1}^N (Y_{\vec{\theta}}(t_i) - y_i)^2 \quad (7)$$

The parameter vector is estimated by using PSO algorithm to minimize the objective function or $\min_{\vec{\theta}} F(\vec{\theta})$.

RESULT AND DISCUSSION

Throughout the computational investigation we choose hyper-parameters $w_{min} = 0.4$, $w_{max} = 0.9$, $c_1 = c_2 = 2$. Whilst the number of observed particles N was varied to examine its effects on the model.

In this investigation we restrict the parameters space with the following bounds

$$\begin{aligned} y_0 &= [16195, 16720] \\ y_{max} &= [48564, 49504] \\ C &= (0, 3] \\ D &= (0.0, 1] \\ E &= [60, 100] \end{aligned}$$

The boundaries for y_0 , y_{max} and E are obtained by guessing from the data. Meaning that the values is from the minimum and maximum values of the data respectively and infer that the inflection point lies between 60 and 100. Meanwhile the parameters boundaries for C and D are obtained by trial-and-error to form a curve that relatively matches the data. The exact values of those parameters which minimize the squared-error will be obtained by using PSO.

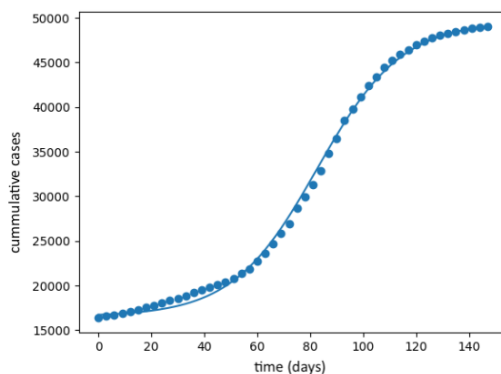


Figure 1. Comparison of fitted model (solid line) and observed data (dotted line)

The fitted model obtained from PSO algorithm is depicted in Figure 1. As seen that the model approximates the data relatively close. From the estimation performed using PSO, the obtained values are as follows:

$$\begin{aligned}
 y_0 &= 16531.20376, \\
 y_{max} &= 49612.66269, \\
 C &= 0.06272, \\
 D &= 0.062726, \\
 E &= 68.06959.
 \end{aligned}$$

Though the model fails to approximate the linear trend in time range between 20 to 40, the model performs a good approximate subsequently. The outcome is as such because this model relies entirely on exponential behavior. Meaning that the growth the growth that can be captured is the growth with an exponential trend. Furthermore, this model can capture the phenomenon of slow case increases. Analytically, this is demonstrated by a change in curvature after passing through the inflection point namely E . This parameter is crucial because, after this point $t > E$, the increase in cases is not as rapid as before. In other words, the cases have already passed their peak.

The PSO algorithm requires not only multiple initial particles, but also its randomness. Five different simulations have been conducted for each number of initial particles, namely $N=20$, $N=50$ and $N=100$. The result can be seen in Table 1, Table 2, and Table 3. The simulation is limited to that extent because as the number of particles increases, the computational time increases. This also applies to the number of iterations. This limitation arises due to the computational complexity of handling a larger dataset (parameters and data) and performing more iterations. It's a trade-off between the precision of results and the time and resources required for the simulations.

Table 1. $N = 20$

No	(y_0, y_{max}, C, D, E)
1	(16337.65, 49546.18, 1.8852, 0.06251, 72.37518)
2	(16411.20, 49186.11213, 1.3002, 0.06292, 78.05831)
3	(16477.81, 49414.91646, 2.2393, 0.06565, 70.09995)

4	(16384.59, 49447.13700, 2.9412, 0.06411, 65.77827)
5	(16313.97643, 49713.68778, 3.7805, 0.06330, 62.46834)

Table 2. $N = 50$

No	(y_0, y_{max}, C, D, E)
1	(16394.45242, 49694.88426, 1.47501, 0.06062, 76.51574)
2	(16411.20331, 49186.11213, 1.30029, 0.06292, 78.05831)
3	(16492.98839, 49472.64796, 0.76068, 0.06258, 86.93336)
4	(16460.73434, 49500.44756, 1.06040, 0.06351, 81.84220)
5	(16605.47971, 49222.66639, 1.91524, 0.06395, 71.84526)

Table 3. $N = 100$

No	(y_0, y_{max}, C, D, E)
1	(16784.52, 49686.38940, 3.7976, 0.06578, 63.60088)
2	(16607.87, 49732.49894, 3.28649, 0.06192, 63.51012)
3	(16535.57, 49602.03244, 0.9976, 0.06546, 83.16428)
4	(16541.78, 49981.81336, 3.0673, 0.06275, 65.70096)
5	(17003.47, 49372.94053, 2.4268, 0.06893, 70.70011)

The MAPE obtained from different number of particles are tend to decreases as the number of particles increases. The simulation yields the average MAPE 1.912, 1.776, and 1.594 for $N=20$, $N=50$, $N=100$ respectively.

Previous studies have demonstrated the effectiveness of PSO and its variants in fitting epidemic models. Ma et al. (2022) developed an improved quantum-behaved PSO (QPSO) to estimate parameters of an extended SEAIQRD model for COVID-19 transmission and showed that improved swarm diversity significantly enhances convergence accuracy compared to standard PSO and genetic algorithms. Similarly, Sa'adah et al. (2024) compared GA and PSO for SEIR model parameter estimation using COVID-19 data from

Jakarta and found that PSO achieved lower numerical errors and better adaptation to rapidly changing epidemic trends.

Compared with these studies, the present work applies a bounded empirical PSO scheme where parameter search intervals are constrained by observed data ranges. This ensures interpretable epidemic curvature estimation while maintaining convergence efficiency under smaller swarm sizes. Therefore, the novelty of this study lies in empirically constrained PSO optimization for nonlinear epidemic growth modeling.

CONCLUSION

In conclusion, this study has demonstrated the implementation of the Particle Swarm Optimization (PSO) algorithm for estimating the parameters of logistic curve. The curve is chosen because it offers a fundamental way to understand complex growth dynamics of COVID-19 pandemic. The parameter is estimated by defining an objective function in the form of mean squared error between the resulting model and the data.

Since the algorithm depends on the number of particles and random initialization, we perform optimization by varying the number of initial particles across several experiments. Instead of mean-squared error, we use Mean Absolute Percentage Error (MAPE) to evaluate the fitted model in each experiment. The exploration of random initial values has unveiled a notable trend in MAPE convergence, with values approaching one another as the randomization effect diminishes. Additionally, the relationship between iteration numbers and MAPE indicates a desirable trend wherein increased iterations correspond to reduced MAPE, highlighting the significance of optimization refinements through

iterative processes. Through these collective findings, this study not only validates the utility of PSO in enhancing parameter estimation accuracy but also contributes valuable insights into the interplay between randomization, iteration, and the quality of curve fitting within the context of logistic growth modeling.

Further studies could explore how adaptive or hybrid versions of PSO may improve both convergence stability and computational efficiency. For example, self-adjusting inertia weights or dynamic acceleration coefficients could better manage the trade-off between exploration and exploitation, potentially reducing sensitivity to random initialization. Another open problem worth investigating involves extending this deterministic optimization to include stochastic or data-driven components, allowing the model to account for uncertainty or structural changes in the observed data. Such future explorations would provide a more comprehensive understanding of PSO's capabilities and limitations in modeling complex nonlinear growth phenomena.

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