

Comparison of the Newton–Raphson and Secant Methods in a Simple Pendulum Model

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ABSTRACT

Various problems in mathematics and physics, including the nonlinear pendulum model, cannot be solved analytically, so numerical methods are used to obtain approximate solutions with a certain error tolerance. This study compares the Newton–Raphson and Secant methods in solving nonlinear equations in a pendulum system based on iteration count, error, and convergence stability using a comparative quantitative approach. The results show that neither method is absolutely superior, as both successfully produced approximate solutions. The value of θ (angular displacement) decreases as the pendulum length (L) increases due to the proportional relationship involving potential energy and the factor mgL . The Newton–Raphson method reached the solution in 4 iterations, while the Secant method required 4–6 iterations. The average order of convergence for Newton–Raphson approaches $p \approx 2$ (quadratic), whereas the Secant method approaches $p \approx 1.62$ (superlinear). The differences between the two methods are more influenced by the choice of initial guesses and the respective mechanisms of each method.

Keywords: Convergence Order, Newton-Raphson, Nonlinear Equation, Secant Method, Simple Pendulum Model

INTRODUCTION

In various fields of science, many problems cannot be solved directly using analytical methods, which rely on established algebraic or mathematical formulas (Rohmawati et al., 2025). Complex mathematical problems are often difficult to solve analytically, even though solutions can still be obtained through the use of numerical methods (Pandia & Sitepu, 2021). In numerical methods, calculations are performed iteratively to obtain values that approximate the true result, with the difference referred to as the error (Toha et al., 2025). The smaller the error value, the higher the accuracy of the result.

Physical phenomena in nature are modeled using mathematical equations that represent the relationships between

physical quantities (Ali et al., 2024). Under certain conditions, these relationships are not linear, resulting in nonlinear equations (Inderjeet & Bhardwaj, 2025). One physical model that exhibits nonlinear behavior is the pendulum system. The mathematical equation model of a simple pendulum according to Baker & Blackburn, (2005) can be written as Equation (1). For numerical analysis purposes, the equation can be transformed into a nonlinear algebraic equation using the conservation of energy approach. This results in the form $\cos \theta - C_L = 0$, where C_L is a constant that depends on the parameter L (string length). The root of this equation is then determined using the Newton–Raphson and Secant methods. The Newton–Raphson and Secant methods

are iterative techniques widely used to determine the roots of nonlinear equations because they are relatively simple and efficient. Both methods have their own characteristics and advantages. In finding the roots of nonlinear equations, the Newton–Raphson method is known for its ability to produce highly accurate solutions with a fast convergence rate (Khanal, 2025). Theoretically, the Newton–Raphson method has quadratic convergence, whereas the Secant method has superlinear convergence and therefore tends to be slower (Agustini & Gunawan, 2024).

Although the Newton–Raphson method is generally faster, the Secant method can show better performance for certain curve shapes. The Newton–Raphson method may be more sensitive to the initial guess and the characteristics of the function, whereas the Secant method is relatively more stable under some conditions (Aboamemah et al., 2020). In addition, the Secant method does not require the derivative of the function, making it simpler to implement and computationally more efficient (Dixit & Mathur, 2021; Khanal, 2025). Thus, both methods have different advantages depending on the equation being solved. This is consistent with the study by Gorgey et al. (2024), which shows that each method can outperform the other in different real-world cases.

Many studies have compared the Newton–Raphson and Secant methods. However, most comparative studies are conducted using abstract mathematical test functions that do not represent real physical models. For example, Khanal (2025) compared the Newton–Raphson and Secant methods to find the root of the nonlinear equation $f(x) = 2^x + 3x - 2$, which does not directly represent a physical problem. Consequently, such approaches may not fully capture the performance of these methods when

applied to nonlinear physical systems, such as the pendulum.

This study aims to compare the Newton–Raphson and Secant methods in solving nonlinear equations in a pendulum system model. The comparison is based on the number of iterations, the level of error, and the stability of convergence. This study contributes by applying the comparison within a physical system model rather than an abstract model. Furthermore, it analyzes the empirical order of convergence in relation to its theoretical value, providing a more comprehensive evaluation of the methods.

MATERIAL AND METHOD

This study employs a comparative quantitative approach aimed at comparing the performance of the Newton–Raphson method and the Secant method in determining the roots of nonlinear equations derived from the physical model of a simple pendulum (Amruddin et al., 2022). The initial stage of the study was conducted by formulating the basic equation of a nonlinear pendulum based on a relevant physical model, and then transforming it into the form of a nonlinear root equation $f(\theta) = 0$. The equation is then solved using the two numerical methods being compared.

The equation of motion of a simple pendulum, given by

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta = 0 \quad (1)$$

To apply the Newton–Raphson and Secant methods, the pendulum equation is first transformed into the root equation form $f(\theta) = 0$ by utilizing the law of conservation of mechanical energy. In an ideal pendulum system, the mechanical energy is the sum of kinetic energy and potential energy, whose value remains constant (Walker, 2022), therefore

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta \quad (2)$$

$$\frac{d^2\theta}{dt^2} \frac{d\theta}{dt} = -\frac{g}{L}\sin\theta \frac{d\theta}{dt} \quad (3)$$

$$\frac{d}{dt} \left(\frac{1}{2} \left(\frac{d\theta}{dt} \right)^2 \right) = \frac{d}{dt} \left(\frac{g}{L} \cos \theta \right) \quad (4)$$

$$\frac{d}{dt} \left(\frac{1}{2} \omega^2 \right) = \frac{d}{dt} \left(\frac{g}{L} \cos \theta \right) \quad (5)$$

$$\frac{d}{dt} \left(\frac{1}{2} \omega^2 - \frac{g}{L} \cos \theta \right) = 0 \quad (6)$$

$$\frac{1}{2} \omega^2 - \frac{g}{L} \cos \theta = C \quad (7)$$

$$mL^2 \left(\frac{1}{2} \omega^2 - \frac{g}{L} \cos \theta \right) = \text{konstan} \quad (8)$$

which gives,

$$\frac{1}{2} mL^2 \omega^2 + mgL(1 - \cos \theta) = E \quad (9)$$

In the Equation (9), the parameter values are defined as follows:

Length of the string (L) = {0.5, 1, 2} m

Earth's gravity acceleration (g) = 9.8 m/s

Energy (E) = 2 Joule

Mass of the object (m) = 1 kg

Angular acceleration (ω) = 0 rad/s

Next, the parameters above are substituted into the equation. The variation of parameter L is used to analyze the effect of the string length on the value of θ , as well as to examine the stability and performance of the numerical methods under different system conditions. The substitution of the parameters into the equation yields three equations as follows:

By substituting $\omega = 0$, the equation is obtained

$$mgL(1 - \cos \theta) = E \quad (10)$$

$$\cos \theta = 1 - \frac{E}{mgL} \quad (11)$$

Let $C_L = 1 - \frac{E}{mgL}$, then

$$\cos \theta = C_L \quad (12)$$

$$f(\theta) = \cos \theta - C_L = 0 \quad (13)$$

Substituting the parameter values $m = 1$, $g = 9.8$, and $E = 2$, the following equations are obtained

For L= 0.5

$$f(\theta) = \cos \theta - 0.5918 = 0 \quad (14)$$

For L=1

$$f(\theta) = \cos \theta - 0.7959 = 0 \quad (15)$$

For L=2

$$f(\theta) = \cos \theta - 0.8979 = 0 \quad (16)$$

Equations (14), (15), and (16) are then solved using the Newton–Raphson and Secant methods.

The solution using the Newton–Raphson method is carried out based on the formula $x_{r+1} = x_r - \frac{f(x_r)}{f'(x_r)}$, with $f'(x_r) \neq 0$ (Maharani & Suprpto, 2018). The solution using the Secant method is carried out based on the formula $x_{r+1} = x_r - \frac{f(x_r)(x_r - x_{r-1})}{f(x_r) - f(x_{r-1})}$. In the Newton–Raphson and Secant methods, the variable x is used as a general variable in solving nonlinear equations. In the context of this study, the variable x represents the angular displacement θ .

The collected data consist of the number of iterations required to reach a specified error tolerance, the final absolute error value, and the pattern of convergence stability under various system conditions, including variations in initial guesses and pendulum parameters. Based on Zakaria & Muharramah (2023), the iteration stopping criterion of numerical methods is determined by the formula $|x_{n+1} - x_n| \leq Tol$. In this study, the iteration is stopped when $|\theta_{n+1} - \theta_n| < 0.0001$.

The data are then analyzed using a comparative quantitative approach by comparing the average number of iterations, the magnitude of the error, and the consistency of convergence for both methods. Through this analysis, the method that is more efficient and stable in solving the nonlinear equation of the pendulum model under the tested conditions can be identified.

RESULT AND DISCUSSION

Graphical Representation of The Given Equation

The graphs of the three equations are presented in the following figure

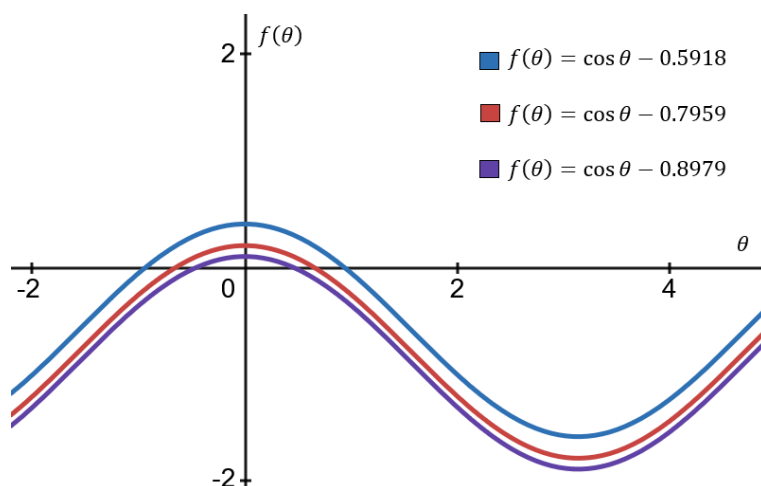


Figure 1. Graph of the Nonlinear Pendulum Equation

Based on Figure 1, the root value of θ obtained using the Newton–Raphson and Secant methods is located at the intersection point of each graph with the θ -axis. As the length of the pendulum string increases, the obtained θ root value decreases.

Result for $f(\theta) = \cos \theta - 0.5918$

The initial guess for the Newton–Raphson method is taken as $\theta_1 = 0.5$, while for the Secant method the initial guesses are taken as $\theta_0 = 0.5$ and $\theta_1 = 1$. The iteration results for both methods are shown in the table below.

Table 1. Iteration Results of the Newton-Raphson Method

Iteration	θ_n	$f(\theta_n)$	$f'(\theta_n)$	$\theta_n + 1$	Error $ \theta_{n+1} - \theta_n $	Status (Error < 0.0001)
1	0.5	0.285783	-0.479426	1.096094	0.596094	Continue
2	1.096094	-0.134726	-0.889429	0.944619	0.151475	Continue
3	0.944619	-0.005748	-0.810274	0.937525	0.007094	Continue
4	0.937525	-0.000015	-0.806096	0.937506	0.000018	Stop

Table 2. Iteration Results of the Secant Method

Iter	θ_{n-1}	θ_n	$f(\theta_n)$	$f(\theta_{n-1})$	$\theta_n - \theta_{n-1}$	θ_{n+1}	Error $ \theta_{n+1} - \theta_n $	Status (Error < 0.0001)
1	0.500000	1.000000	-0.051498	0.285783	0.500000	0.923657	0.076343	Continue
2	1.000000	0.923657	0.011106	-0.051498	-0.076343	0.937201	0.013544	Continue
3	0.923657	0.937201	0.000246	0.011106	0.013544	0.937508	0.000307	Continue
4	0.937201	0.937508	-0.000001	0.000246	0.000307	0.937506	0.000002	Stop

Based on Tables 1 and 2, both methods obtain a root value of $\theta \approx 0.937506$ rad. The Secant method appears to perform slightly better than the Newton–Raphson method. Both methods approach the root value by the fourth iteration. This is evident from the fact that both methods reach the stopping

criterion (0.0001) at that iteration. However, the Secant method shows a smaller error, approximately 0.000002, while the Newton–Raphson method yields an error of approximately ≈ 0.000018 at the fourth iteration.

Result for $f(\theta) = \cos \theta - 0.7959$

The initial guess for the Newton–Raphson method is taken as $\theta_1 = 1$, while for the Secant method the initial guesses

are taken as $\theta_0 = 1$ and $\theta_1 = 1.5$. The results of the iterations of both methods are presented in the following table

Table 3. Iteration Results of the Newton-Raphson Method

Iteration	θ_n	$f(\theta_n)$	$f'(\theta_n)$	$\theta_n + 1$	Error $ \theta_{n+1} - \theta_n $	Status (Error <0.0001)
1	1	-0.255598	-0.841471	0.696249	0.303751	Continue
2	0.696249	-0.028647	-0.641344	0.651582	0.044667	Continue
3	0.651582	-0.000775	-0.606445	0.650305	0.001278	Continue
4	0.650305	-0.000001	-0.605429	0.650304	0.000001	Stop

Table 4. Iteration Results of the Secant Method

Iter	θ_{n-1}	θ_n	$f(\theta_n)$	$f(\theta_{n-1})$	$\theta_n - \theta_{n-1}$	θ_{n+1}	Error $ \theta_{n+1} - \theta_n $	Status (Error <0.0001)
1	1.000000	1.500000	-0.725163	-0.255598	0.500000	0.727836	0.772164	Continue
2	1.500000	0.727836	-0.049284	-0.725163	-0.772164	0.671531	0.056305	Continue
3	0.727836	0.671531	-0.013030	-0.049284	-0.056305	0.651295	0.020236	Continue
4	0.671531	0.651295	-0.000600	-0.013030	-0.020236	0.650317	0.000977	Continue
5	0.651295	0.650317	-0.000008	-0.000600	-0.000977	0.650304	0.000014	Stop

Based on Tables 3 and 4, both methods obtain a root value of $\theta \approx 0.650304$ rad. The Newton–Raphson method appears to be superior in terms of the number of iterations and the magnitude of the error. The Newton–Raphson method requires four iterations to approach the root value (≈ 0.650304), whereas the Secant method requires five iterations. This is evident from the fact that both methods reach the stopping criterion (0.0001) at those iterations. The final error produced by the Newton–Raphson method is ≈ 0.000001 , while the

Secant method yields ≈ 0.000014 . Therefore, the Newton–Raphson method demonstrates a smaller error than the Secant method.

Result for $f(\theta) = \cos \theta - 0.8979$

The initial guess for the Newton–Raphson method is taken as $\theta_1 = 2$, while for the Secant method the initial guesses are taken as $\theta_0 = 2$ and $\theta_1 = 2.5$. The results of the iterations of both methods are presented in the following table.

Table 5. Iteration Results of the Newton-Raphson Method

Iteration	θ_n	$f(\theta_n)$	$f'(\theta_n)$	$\theta_n + 1$	Error $ \theta_{n+1} - \theta_n $	Status (Error <0.0001)
1	2	-1.314047	-0.909297	0.554877	1.445123	Continue
2	0.554877	-0.047935	-0.526839	0.463891	0.090985	Continue
3	0.463891	-0.003582	-0.447432	0.455886	0.008005	Continue
4	0.455886	-0.000029	-0.440258	0.455821	0.000065	Stop

Table 6. Iteration Results of the Secant Method

Iter	θ_{n-1}	θ_n	$f(\theta_n)$	$f(\theta_{n-1})$	$\theta_n - \theta_{n-1}$	θ_{n+1}	Error $ \theta_{n+1} - \theta_n $	Status (Error <0.0001)
1	2.000000	2.500000	-1.699044	-1.314047	0.500000	0.293431	2.206569	Continue
2	2.500000	0.293431	0.059357	-1.699044	-2.206569	0.367917	0.074485	Continue
3	0.293431	0.367917	0.035179	0.059357	0.074485	0.476290	0.108373	Continue
4	0.367917	0.476290	-0.009198	0.035179	0.108373	0.453827	0.022463	Continue
5	0.476290	0.453827	0.000876	-0.009198	-0.022463	0.455780	0.001953	Continue
6	0.453827	0.455780	0.000018	0.000876	0.001953	0.455821	0.000041	Stop

Based on Tables 5 and 6, both methods obtain a root value of $\theta \approx 0.455821$ rad. The Newton–Raphson method appears to be superior in terms of the number of iterations. The Newton–Raphson method requires four iterations to approach the root value (≈ 0.455821), whereas the Secant method requires six iterations. This is evident from the fact

that both methods reach the stopping criterion (0.0001) at those iterations. However, the final error produced by the Secant method (≈ 0.000041) is smaller than that of the Newton–Raphson method (≈ 0.000065).

Comparison of the Newton–Raphson and Secant Methods.

Table 7. Comparison of the Newton–Raphson and Secant Methods

L	Method	Iteration	Computed Root	Final Error
0.5	Newton-Raphson	4	0.937506	0.000018
0.5	Secant	4	0.937506	0.000002
1	Newton-Raphson	4	0.650304	0.000001
1	Secant	5	0.650304	0.000014
2	Newton-Raphson	4	0.455821	0.000065
2	Secant	6	0.455821	0.000041

Overall, there is no method that is absolutely superior. The Newton–Raphson method tends to require fewer iterations, whereas the Secant method in some cases produces a smaller final error. The Newton–Raphson method has a quadratic order of convergence, but it depends on the derivative of the function. In contrast, the Secant method does not use derivatives, so under certain parameters it can exhibit better stability,

although its order of convergence is lower.

The order of convergence of both methods is determined using the formula $p = \frac{\ln(\frac{e_{n+1}}{e_n})}{\ln(\frac{e_n}{e_{n-1}})}$ (Grau-Sanchez et al., 2011).

The e_n represents the error (e) at the n-th iteration. The error data obtained from equation $f(\theta) = \cos \theta - 0.5918$ are used as a representation of the order of convergence of the two methods. This is presented in the following table.

Table 8. Average Order of Convergence of the Newton–Raphson and Secant Methods

Iteration Data	Newton-Raphson	Secant
1,2,3	2.234468	2.189827
2,3,4	1.952401	1.329258
Average	2.093434	1.759542

Based on Table 8, the calculation of the order of convergence is performed using iteration data (1, 2, 3) and (2, 3, 4), which produce values p_1 and p_2 , respectively. These two values are not identical because they are derived from different combinations of error data. This difference is expected, especially in the early iterations where the error values still fluctuate. To obtain a more stable and representative estimate of the order of convergence, the average of these two values is used. This approach aims to reduce the influence of early iteration fluctuations and provide a more accurate representation of the convergence behavior of the method.

The calculation results show that the error of the Newton method decreases

quadratically, indicated by the average order of convergence approaching $p \approx 2$, where the error reduction becomes very rapid once the iteration approaches the root. In contrast, the Secant method produces an average order of convergence approaching $p \approx 1.62$, which is consistent with the superlinear convergence theory (Sauer, 2012), with a slower decrease in error and fluctuations during the early iterations because this method does not use derivative information. Thus, the Newton method is superior in terms of convergence speed, whereas the Secant method is computationally simpler since it does not require the derivative of the function.

The convergence speed of both methods is shown in the following figure.

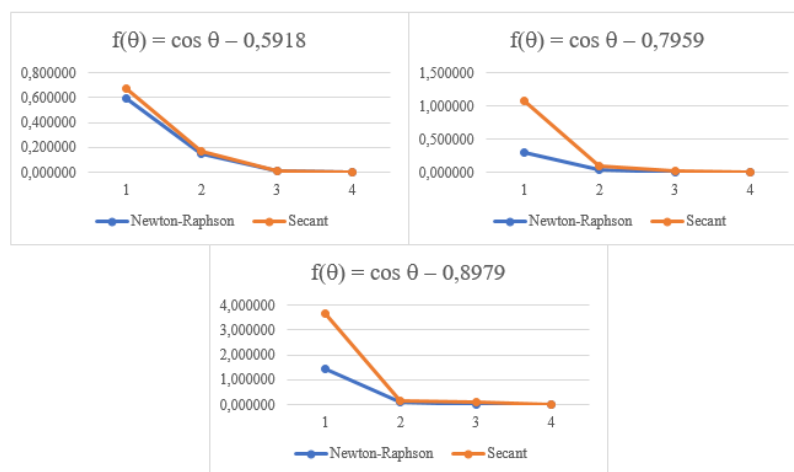


Figure 2. Convergence Speed of the Newton–Raphson and Secant Methods

Based on Figure 2, the x-axis represents the iteration number, while the y-axis represents the magnitude of the error. In all three graphs, the errors of both methods decrease drastically within 2–3 iterations, and the errors approach zero after the third iteration. This indicates that both methods successfully find the root quickly for this function. The initial error of the Newton–Raphson method appears smaller and its decrease is smoother compared to the Secant method. This occurs because the Secant method uses a gradient approximation

from two initial points. If the two initial guesses are not close to the root, the initial error becomes larger (Canale & Chapra, 2015). Therefore, the Secant method is more sensitive to the choice of initial guesses.

The function $f(\theta) = \cos \theta - C_L$ is a smooth function with a derivative $f'(\theta) = -\sin \theta$ that has a sufficiently large value around the root, so the convergence condition of the Newton–Raphson method is satisfied (Burden & Faires, 2011; Imron et al., 2022). Therefore, both methods show rapid

convergence. The difference in the number of iterations is mainly influenced by the choice of the initial guesses and the approximation mechanism of each method, rather than by the presence of a singularity or a derivative that approaches zero. Thus, both methods can be used to solve nonlinear equations in the pendulum model.

The calculation results show that the maximum oscillation angle (θ) decreases as the string length (L) increases. Physically, this occurs because the potential energy of a pendulum is proportional to the factor mgL . For the same total energy, an increase in the string length causes the energy required to reach a certain angle to become larger, so the system can only reach a smaller maximum angle.

CONCLUSION

This study aims to compare the performance of the Newton–Raphson and Secant methods in determining the roots of nonlinear equations in a simple pendulum model based on the number of iterations, error level, and convergence characteristics. The results show that both methods are effective, with the Newton–Raphson method converging faster due to its quadratic order (≈ 2), while the Secant method has a superlinear order (≈ 1.62) with a slightly slower convergence rate. Neither method is absolutely superior, as accuracy depends on parameter conditions. Newton–Raphson is more efficient when derivatives are available, whereas the Secant method is simpler without requiring derivatives yet remains competitive. Thus, the choice of method depends on the function’s characteristics and the desired computational efficiency. This study confirms that both methods are valid for physical systems, although it is limited to a simplified model.

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