

Fuzzy Graph Coloring for Traffic Signal Phase Optimization at Adjacent Urban Intersections

Sri Puspita Dewi I Yado*, Nurwan, Djihad Wungguli, Nisky Imansyah Yahya,
Armayani Aarsal

*e-mail: dewiyado3@gmail.com

*Program Study of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Negeri
Gorontalo, Gorontalo, Indonesia*

ABSTRACT

Traffic congestion at adjacent intersections is a critical urban mobility problem as vehicle queues and turning conflicts directly influence nearby performance. Conventional fixed-phase systems often fail to accommodate heterogeneous and fluctuating flows, especially when intersections are treated independently. This study aims to optimize traffic light phase arrangements at two adjacent intersections on Prof. Dr. Jhon Aryo Katili Road, Gorontalo, using a fuzzy graph coloring approach. Vehicle flow data were collected during morning and afternoon peak periods and converted into Passenger Car Equivalent (PCE) values. Each traffic movement was represented as a vertex, while conflicting movements were modeled as edges in a fuzzy graph. Triangular membership functions classified traffic intensity, followed by α -cut-based fuzzy graph coloring to determine feasible signal phases. The results yield a chromatic number of eight, representing eight traffic light phases. This indicates that fuzzy graph coloring provides a more conflict-sensitive and adaptive structure than the existing three-phase system. The main contribution lies in extending fuzzy graph optimization from isolated to adjacent intersection settings, providing a mathematical basis for more efficient urban traffic management.

Keywords: Fuzzy graph, fuzzy graph coloring, chromatic number, traffic light optimization, adjacent intersections.

INTRODUCTION

According to Pobela dkk (2020) Urban traffic congestion has become an increasingly complex problem in many cities, particularly in road networks where traffic demand grows faster than the capacity of road infrastructure and signal control systems. Intersections represent critical points in urban transportation systems because they accommodate multiple directional movements, turning flows, and potential conflicts within a limited spatial area. The problem becomes more complex when intersections are located close to each other, since queues and delays at one intersection may propagate to the

next intersection and disturb the overall performance of the corridor. Therefore, traffic light regulation at adjacent intersections requires a modeling approach that can represent not only individual vehicle movements but also conflict relations among movements across nearby intersections.

Fuzzy graph theory provides a relevant mathematical framework for analyzing traffic light systems under uncertainty. In a classical graph, vertices and edges are generally represented in a crisp manner, whereas in a fuzzy graph each vertex or edge may have a membership degree that reflects the intensity or strength of its presence. This

characteristic is suitable for traffic modeling because vehicle flows are not static; they vary by time period, road approach, vehicle type, and movement direction. In the context of traffic lights, each vehicle movement can be represented as a vertex, while conflict relations among movements can be represented as edges. By assigning fuzzy membership values to traffic movements, the model can classify vehicle flow intensity into linguistic categories such as low, medium, and high, allowing traffic signal phases to be determined more flexibly than in purely deterministic models.

The specific problem addressed in this study concerns the traffic light arrangement at two adjacent intersections on Prof. Dr. Jhon Aryo Katili Road, Gorontalo. Based on the submitted draft, the existing traffic light system applies a three-phase arrangement, while the observed traffic movements involve twenty-four nodes distributed across two neighboring intersections. Field observations show that vehicle flows differ substantially among movement directions during morning and afternoon periods. Some movements have relatively high traffic intensity, while others are lower or moderate. Such heterogeneity indicates that the existing fixed phase system may not adequately accommodate variations in traffic demand and conflict patterns. Consequently, a more systematic mathematical model is required to determine traffic light phases that can minimize movement conflicts and improve phase allocation at adjacent intersections.

Previous studies have demonstrated that fuzzy graph approaches are effective for traffic light optimization and traffic flow analysis. Kurniawan (2017) employed a fuzzy graph approach combined with max-plus algebra to model a traffic light control system at a road intersection. The results

showed that the integration of these two approaches could effectively represent signal phase synchronization and mathematically analyze traffic signal cycle times. However, the study was limited to a single intersection and did not consider variations in traffic flow intensity through fuzzy membership degrees or interactions between adjacent intersections.

Furthermore, Wahyu et al. (2023) applied a Mamdani fuzzy inference system based on fuzzy graphs to optimize traffic signal control at an urban intersection. The study used traffic volume data as input and produced signal phase durations that were more adaptive to traffic conditions than a fixed-time control system. Nevertheless, the proposed approach primarily focused on fuzzy inference for determining signal durations and did not utilize fuzzy graph coloring to group non-conflicting traffic movements.

Ramadhani and Munandar (2024) investigated fuzzy graph structures and their mathematical properties, along with their applications in decision-making problems. Their findings indicated that fuzzy graphs provide a more realistic representation of uncertain relationships than classical graphs. However, the study was mainly theoretical and did not implement fuzzy graph models in transportation systems or traffic signal optimization.

A more application-oriented study was conducted by Sijabat et al. (2025), who applied fuzzy graph coloring to optimize traffic signal phases and reduce vehicle waiting times at the Digulis Roundabout in Pontianak. The researchers constructed a conflict graph based on conflicting traffic movements and applied the α -cut concept to determine the chromatic number, representing the minimum number of traffic signal phases required. The results demonstrated that the proposed approach

generated a more efficient phase arrangement than the existing system, thereby reducing potential traffic conflicts. However, the model was still limited to a single intersection and did not consider signal interactions among adjacent intersections.

Similarly, Sugarda (2025) employed fuzzy graph modeling and graph coloring to analyze traffic flow at the Jamin Ginting Flyover intersection. Traffic volume data were converted into fuzzy membership values, followed by the construction of a conflict graph and the application of graph coloring to determine traffic signal phases. The results indicated that the fuzzy graph approach produced signal phase arrangements that better reflected the conflict levels among traffic movements, thereby improving traffic signal efficiency. Nevertheless, the study also focused on a single intersection and did not investigate signal coordination within a network of adjacent intersections.

Based on these previous studies, it can be concluded that fuzzy graph approaches are effective in modeling traffic conflicts, determining traffic signal phases, and improving the efficiency of traffic signal control. However, several limitations remain: (1) most studies analyze only a single isolated intersection; (2) they do not model conflict interactions among traffic movements at adjacent intersections; (3) they do not integrate empirical traffic volume data from multiple intersections into a unified conflict graph; and (4) they do not investigate how traffic signal coordination between adjacent intersections can be achieved using a fuzzy graph coloring approach.

These limitations reveal a significant research gap that motivates the present study. Therefore, this research develops a fuzzy graph coloring model for two adjacent intersections on Prof. Dr. Jhon Aryo Katili Road, Gorontalo.

Unlike previous studies, the proposed model integrates traffic volume data from two neighboring intersections, converts them into fuzzy membership degrees using triangular membership functions, constructs a unified traffic conflict graph, and applies the α -cut concept together with fuzzy graph coloring to determine the minimum number of conflict-free traffic signal phases.

The objective of this study is to develop and apply a fuzzy graph coloring model to determine traffic light phases at two adjacent intersections on Prof. Dr. Jhon Aryo Katili Road, Gorontalo. More specifically, this study aims to classify traffic movement intensity using fuzzy membership functions, construct a fuzzy conflict graph for adjacent intersections, and determine the chromatic number as the basis for traffic signal phase optimization.

MATERIAL AND METHOD

This research was conducted at two adjacent intersections on Prof. Dr. Jhon Aryo Katili Road, namely the Gorontalo Traditional House intersection and the BRI Andalas intersection, Gorontalo City. Traffic data were collected on Monday during the morning peak hour (08:00–09:00) and the afternoon peak hour (15:30–16:30). The research methods included a literature review, field observation, and interviews. The literature review was conducted by examining journals, books, and other relevant references related to fuzzy graph theory and fuzzy graph coloring, while field observations and interviews were carried out to obtain traffic volume data and information on the existing traffic signal system.

This study employed a fuzzy graph approach to optimize traffic signal phases at the two adjacent intersections. The research procedure began with collecting traffic volume data, which were then converted into Passenger Car

Units (PCU). The converted data were used to calculate fuzzy membership degrees using triangular membership functions. Subsequently, a fuzzy graph was constructed by representing traffic movements as vertices and conflict relationships as edges. The α -cut concept was then applied to generate fuzzy graphs at each membership level, followed by the fuzzy graph coloring process to determine the chromatic number. Finally, the obtained chromatic number was used as the basis for determining the optimal traffic signal phases and was compared with the existing traffic signal system.

The research procedure is illustrated in the flowchart below.

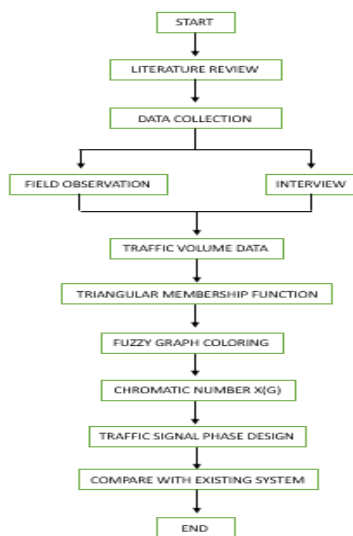


Figure 1. Flowchart of the Research Process

The data analysis procedure applied can be described as follows:

1. Himpunan Fuzzy

One of the methods is the triangular fuzzy membership function, which is a function with three parameters $a, b, c \in \mathbb{R}$, where $a < b < c$, expressed as follows :

$$\mu(x) = \begin{cases} 0, & x \leq a \\ \frac{x-a}{b-a}, & a \leq x < b \\ \frac{c-x}{c-b}, & b \leq x < c \\ 0, & x \geq c \end{cases} \quad (1)$$

The graph of the triangular membership function is shown in Figure 1.

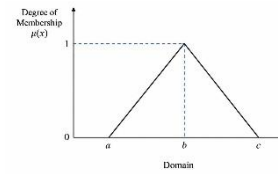


Figure 2. Graphical representation of the triangular curve.

Fuzzy sets have three basic operators used to combine fuzzy sets, namely:

1. AND Operator (intersection)

The AND operator corresponds to the intersection operation on fuzzy sets $\tilde{A}$ and $\tilde{B}$, denoted as $\tilde{A} \cap \tilde{B}$. Its membership function is defined as:

$$\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x)) \quad (2)$$

2. OR Operator (union)

The OR operator relates to the union operation on fuzzy sets $\tilde{A}$ and $\tilde{B}$, denoted as $\tilde{A} \cup \tilde{B}$. Its membership function is defined as:

$$\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x)) \quad (3)$$

3. NOT Operator (complement)

The NOT operator corresponds to the complement operation on a fuzzy set. Its membership function is expressed as:

$$\mu_{A'}(x) = 1 - \mu_A(x) \quad (4)$$

Definition 1 According to Rahayuningsih (2018) a fuzzy set A in a universe X is said to be a subset of a fuzzy set B in X if the degree of membership of every element $x \in X$ in A does not exceed its degree of membership in B , i.e., $\mu_A(x) \leq \mu_B(x), \forall x \in X$. As a note, the membership functions $\mu_A(x)$ and $\mu_B(x)$ can also be written as $A(x)$ and $B(x)$, respectively.

Definition 2 Suppose two fuzzy sets A and B are given on a universe X . The union operation $A \cup B$ and the

intersection operation $A \cap B$ are fuzzy sets on X whose membership degrees for each $x \in X$ are defined as follows (Setiawan & Juniati, 2013):

$$1. \mu_{A \cup B}(x) = \mu_A(x) \vee \mu_B(x) \\ \max(\mu_A(x), \mu_B(x)), \forall x \in X \quad (5)$$

$$2. \mu_{A \cap B}(x) = \mu_A(x) \wedge \mu_B(x) \\ \min(\mu_A(x), \mu_B(x)), \forall x \in X \quad (6)$$

2. Graf Fuzzy

According to Fanani (2016) a graph G is defined as an ordered pair of sets, $G = (V, E)$, where $V(G)$ is a non empty set of vertices and $E(G)$ is a finite set (which may be empty) of edges, each representing an unordered pair of vertices in $V(G)$. Therefore, a graph may have no edges, but it must contain at least one vertex (Kaviya & Ramya, 2022).

A fuzzy graph G is a pair $\tilde{G} = (V, \sigma, \mu)$, where V is a set of vertices. $\sigma: V \rightarrow [0,1]$ assigns a membership degree to each vertex, and $\mu: V \times V \rightarrow [0,1]$ assigns a membership degree to each edge (Sudin, 2024). The following is an example of a fuzzy graph (Arifai & Juniati, 2013).

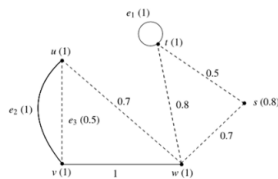


Figure 3. Fuzzy Graph

Definition 3 Let V be a finite and non-empty set of vertices. Then a fuzzy graph $\tilde{G} = (V, \sigma, \mu)$ consists of the set V together with two functions, namely $\sigma: V \rightarrow [0,1]$ and $\mu: V \times V \rightarrow [0,1]$, such that $\mu(u,v) \leq \min\{\sigma(u), \sigma(v)\} \forall u,v \in V$. The fuzzy set σ represents the fuzzy vertex set of G , while the fuzzy set μ represents the fuzzy edge set of G (Ramadhani & Munandar, 2024).

Coloring in fuzzy graphs in the context of traffic management uses the concept from (Sijabat et al., 2025) to

determine the chromatic number of a fuzzy graph. According to their research, the chromatic number of a fuzzy graph is the smallest number k such that every vertex in the fuzzy graph can be colored with k different colors, with the condition that two adjacent vertices do not share the same color.

Definition 4 According to Fuad & Rahadjeng (2014) Let $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \dots, \gamma_k\}$ be a family of fuzzy sets on V . Then Γ is called a fuzzy k -coloring of $\tilde{G} = (V, \sigma, \mu)$ if it satisfies:

1. $\gamma_1 \cup \gamma_2 \cup \gamma_3 \cup \dots \cup \gamma_k = V$
2. $\gamma_i \cap \gamma_j = \emptyset$, for $\gamma_i, \gamma_j, i = 1, 2, 3, \dots, k$ and $j = 1, 2, 3, \dots, k$, is a family of fuzzy graph coloring in which every member consists of vertices that are pairwise non-adjacent.

For a fuzzy graph $\tilde{G} = (V, \sigma, \mu)$ its chromatic number is denoted as $\chi(G) = \{X_\alpha\}$, where χ_α represents the chromatic number of G_α , and its values vary depending on the membership degrees of vertices and edges in the family of fuzzy graphs. In this case, the concept of the α -cut of a fuzzy graph is used, namely $G_\alpha = (V_\alpha, E_\alpha, \mu)$ with $V_\alpha = \{v \in V \mid \sigma_L(v) \geq \alpha\}$, $E_\alpha = \{e \in E \mid \mu_L(e) \geq \alpha\}$.

With $\mu: V \rightarrow I$ and $\alpha \in I$, where $I = \{\tilde{T}, \tilde{S}, \tilde{R}\}$ which respectively denote $\tilde{T}$ (High), $\tilde{S}$ (Medium), and $\tilde{R}$ (Low). The order of the fuzzy graph's chromatic numbers is determined starting from $\alpha = \tilde{T}$, then $\alpha = \tilde{S}$, and finally $\alpha = \tilde{R}$.

RESULT AND DISCUSSION

Prof. Dr. Jhon Aryo Katili Road is a public road and part of a provincial road, serving as a connection between districts and cities and supporting trade activities. This road has implemented a 3-phase traffic light system at two intersections. The following is an illustration of the traffic flow at the intersections of Prof. Dr. Jhon Aryo Katili Road.

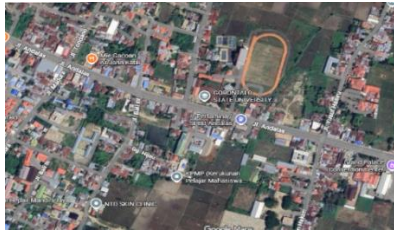


Figure 4. Intersection of Prof. Dr. Jhon Aryo Katili Road

There are as many as 14 nodes at the intersection of the Rumah Adat area and the area around BRI on Jhon Aryo Katili Road, namely A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, and X. The following is information regarding the nodes of traffic flow movement at the intersection of Prof. Dr. Jhon Aryo Katili Road.

Table 1. Vehicle Flow at Two Road Intersections"

Intersection	Approach	Movement	Description
Intersection 1	East	A	Prof. DR. Jhon Aryo Katili Street – Arif Rahman Hakim Street
		B	Prof. DR. Jhon Aryo Katili Street – Prof. DR. Jhon Aryo Katili Street
		C	Prof. DR. Jhon Aryo Katili Street – Yusuf Hasiru Street
	West	D	Prof. DR. Jhon Aryo Katili Street – Yusuf Hasiru Street
		E	Prof. DR. Jhon Aryo Katili Street – Prof. DR. Jhon Aryo Katili Street
		F	Prof. DR. Jhon Aryo Katili Street – Arif Rahman Hakim Street
	North	G	Yusuf Hasiru Street – Prof. DR. Jhon Aryo Katili Street
		H	Yusuf Hasiru Street– Arif Rahman Hakim Street
		I	Yusuf Hasiru Street – Prof. DR. Jhon Aryo Katili Street
	South	J	Arif Rahman Hakim Street – Prof. DR. Jhon Aryo Katili Street
		K	Arif Rahman Hakim Street – Yusuf Hasiru Street
		L	Arif Rahman Hakim Street –Prof. DR. Jhon Aryo Katili Street
Intersection 2	East	M	Prof. DR. Jhon Aryo Katili Street – Madura

Street	
N	Prof. DR. Jhon Aryo Katili Street – Prof. DR. Jhon Aryo Katili Street
O	Prof. DR. Jhon Aryo Katili Street – Tondano Street
P	Prof. DR. Jhon Aryo Katili Street – Tondano Street
Q	Prof. DR. Jhon Aryo Katili Street – Prof. DR. Jhon Aryo Katili Street
R	Prof. DR. Jhon Aryo Katili Street – Madura Street
S	Tondano Street – Prof. DR. Jhon Aryo Katili Street
T	Tondano Street – Madura Street
U	Tondano Street – Prof. DR. Jhon Aryo Katili Street
V	Madura Street – Prof. DR. Jhon Aryo Katili Street
W	Madura Street – Tondano Street
X	Madura Street – Prof. DR. Jhon Aryo Katili Street

Based on field observations, the number of vehicles passing through the intersection of Prof. Dr. Jhon Aryo Katili Road in the Rumah Adat and BRI areas on January 12, 2026, in the morning from 08:00 to 09:00 WIB and in the afternoon from 16:00 to 17:00 WIB is shown in Table 3.

Table 2. Traffic Flow at Two Intersections of Prof. Dr. Jhon Aryo Katili Road from 08:00 – 09:00

Inter section	Approach	Movement	Light Vehicle (LV)	Heavy Vehicle (HV)	Motorcycle (MC)	Total Vehicles / hour
1	East	A	47	4	93	144
		B	178	21	462	661
		C	11	1	70	82
	West	D	10	2	43	55
		E	189	14	447	650
		F	51	4	162	217
	North	G	12	3	46	61
		H	29	4	152	185
		I	13	4	38	55
	South	J	76	3	151	230
		K	11	2	164	177
		L	88	6	182	276
2	East	M	45	4	213	262
		N	191	38	558	787
		O	46	4	142	192

West	P	7	1	47	55
	Q	289	25	681	995
	R	34	2	133	169
North	S	17	4	29	50
	T	34	4	228	266
	U	34	9	159	202
South	V	29	4	146	179
	W	50	2	252	304
	X	37	2	128	167

Table 3. Traffic Flow at Two Intersections of Prof. Dr. Jhon Aryo Katili Road from 15.30 – 16.30

Inter section	Approach	Movement	Light Vehicle (LV)	Heavy Vehicle (HV)	Motorcycle (MC)	Total Vehicles / hour
1	East	A	59	3	163	225
		B	296	19	701	1.016
		C	22	2	91	115
	West	D	11	2	52	65
		E	314	27	688	1.029
		F	43	4	133	180
	North	G	27	7	123	157
		H	58	11	217	286
		I	22	8	106	136
	South	J	71	6	224	301
		K	39	2	121	162
		L	76	5	142	223
2	East	M	68	2	212	282
		N	383	32	736	1.151
		O	57	2	145	204
	West	P	13	1	73	87
		Q	294	25	751	1.070
		R	40	1	138	179
	North	S	48	1	217	266
		T	60	1	314	375
		U	16	1	46	63
	South	V	57	2	214	273
		W	50	4	412	466
		X	42	3	186	231

At the intersection of Prof. Dr. Jhon Aryo Katili Road, there are various types of vehicles with varying specifications. To simplify the calculation, all types of vehicles are converted using the passenger car equivalent (PCE) (Manual Kapasitas Jalan Indonesia, 1997). The Passenger Car Equivalent (PCE) consists of the PCE values for heavy vehicles (HV), light vehicles (LV), and motorcycles (MC) (Yulipriyono & Purwanto 2017).

$$q = q_{LV} \times emp_{LV} + q_{HV} \times emp_{HV} + q_{MC} \times emp_{MC}$$

$$= (47 \times 1) + (4 \times 1,3) + (93 \times 0,4)$$

$$= 89,4 \approx 89 \text{ PCE/hour}$$

Furthermore, by applying the same method as for node A in Table 3 and using the Python application in VS Code, the conversion results for the nodes of traffic flow at the two intersections of Prof. Dr. Jhon Aryo Katili Road can be seen in Table 5 (for the morning) and Table 6 (for the afternoon).

Table 4. Total Traffic Flow from 08:00 – 09:00

Intersection	Approach	Movement	Normal Flow (q).
1	East	A	89
1	East	B	390
1	East	C	40
1	West	D	30
1	West	E	386
1	West	F	121
1	North	G	34
1	North	H	95
1	North	I	33
1	South	J	140
1	South	K	79
1	South	L	169
2	East	M	135
2	East	N	464
2	East	O	108
2	West	P	27
2	West	Q	594
2	West	R	90
2	North	S	34
2	North	T	130
2	North	U	109
2	South	V	93
2	South	W	153
2	South	X	91

Table 5. Total Traffic Flow from 15.30 – 16.30

Intersection	Approach	Movement	Normal Flow (q).
1	East	A	128
1	East	B	601
1	East	C	61
1	West	D	34
1	West	E	624
1	West	F	101
1	North	G	85
1	North	H	159
1	North	I	75
1	South	J	168

1	South	K	90
1	South	L	139
2	East	M	155
2	East	N	719
2	East	O	118
2	West	P	44
2	West	Q	627
2	West	R	96
2	North	S	136
2	North	T	187
2	North	U	36
2	South	V	145
2	South	W	220
2	South	X	120

The road widths for each direction/approach at the two intersections of Prof. Dr. Jhon Aryo Katili Road are as follows: Prof. Dr. Jhon Aryo Katili Road has a width of 14 meters, Arif Rahman Hakim Road 6.34 meters, Yusuf Hasiru Road 5.9 meters, Tondano Road 7.1 meters, and Madura Road 9.1 meters.

The node membership degree (σ) is represented by a curve that describes the membership function, which is grouped into three categories: Low (L), Medium (M), and High (H).

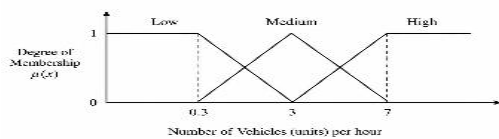


Figure 5. Membership Function Graph of Node (σ)

Figure 4 displays the curves of the three node membership functions used to determine the membership values in the fuzzy set. Based on these curves, the membership functions are divided into three categories: Low (L), Medium (M), and High (H). Accordingly, the equations for the node membership functions can be formulated as follows.

$$\sigma_{Low}(x) \begin{cases} 1, & x \leq 30 \\ \frac{300 - x}{300 - 30}, & 30 < x < 300 \\ 0, & x \geq 300 \end{cases} \quad (1)$$

$$\sigma_{Medium}(x) \begin{cases} 0, & \leq 30 \text{ or } x \geq 700 \\ \frac{x - 30}{300 - 30}, & 30 < x < 300 \\ \frac{700 - x}{700 - 300}, & 300 \leq x \leq 700 \end{cases} \quad (2)$$

$$\sigma_{High}(x) \begin{cases} 0, & x \leq 300 \\ \frac{x - 300}{700 - 300}, & 300 < x < 700 \\ 1, & \geq 700 \end{cases} \quad (3)$$

Based on equations (1) – (3), the node membership degree can be determined as follows, by taking the traffic flow value at node A from 08:00 – 09:00 as $x = 89$ vehicles/hour.

$$\sigma_{(R)}(89) = \frac{300 - 89}{300 - 30} = 0,78$$

$$\sigma_{(S)}(89) = \frac{89 - 30}{300 - 30} = 0,22$$

$$\sigma_{(T)}(80) = 0$$

Using the same method as for node A, the membership degrees for the other fuzzy graph nodes can also be determined, so that the results for 08:00 – 09:00 and 15:30 – 16:30 are presented in the following table.

Table 6. Membership Degrees of Fuzzy Graph Nodes on Prof. Dr. Jhon Aryo Katili Road from 08:00 – 09:00

Intersection	Movement	q	$\sigma(R^-)$	$\sigma(S^-)$	$\sigma(T^-)$
1	A	89	0,78	0,22	0
1	B	390	0	0,78	0,23
1	C	40	0,96	0,04	0
1	D	30	1	0	0
1	E	386	0	0,79	0,21
1	F	121	0,66	0,34	0
1	G	34	0,99	0,01	0
1	H	95	0,76	0,24	0
1	I	33	0,99	0,01	0
1	J	140	0,59	0,41	0

1	K	79	0,82	0,18	0
1	L	169	0,49	0,51	0
2	M	135	0,61	0,39	0
2	N	461	0	0,60	0,40
2	O	108	0,71	0,29	0
2	P	27	1	0	0
2	Q	591	0	0,27	0,73
2	R	90	0,78	0,22	0
2	S	34	0,99	0,01	0
2	T	130	0,63	0,37	0
2	U	109	0,71	0,29	0
2	V	93	0,77	0,23	0
2	W	153	0,54	0,46	0
2	X	91	0,77	0,23	0

Table 7. Membership Degrees of Fuzzy Graph Nodes on Prof. Dr. Jhon Aryo Katili Road from 15.30 – 16.30

Intersection	Movement	q	$\sigma(R^-)$	$\sigma(S^-)$	$\sigma(T^-)$
1	A	128	0,64	0,36	0
1	B	601	0	0,25	0,75
1	C	61	0,89	0,11	0
1	D	34	0,99	0,01	0
1	E	624	0	0,19	0,81
1	F	101	0,74	0,26	0
1	G	85	0,80	0,20	0
1	H	159	0,52	0,48	0
1	I	75	0,83	0,17	0
1	J	168	0,49	0,51	0
1	K	90	0,78	0,22	0
1	L	139	0,60	0,40	0
2	M	155	0,54	0,46	0
2	N	719	0	0	1
2	O	118	0,67	0,33	0
2	P	44	0,95	0,05	0
2	Q	627	0	0,18	0,82
2	R	96	0,76	0,24	0
2	S	136	0,61	0,39	0
2	T	187	0,42	0,58	0
2	U	36	0,98	0,02	0
2	V	145	0,57	0,43	0
2	W	220	0,30	0,70	0
2	X	120	0,67	0,33	0

Next, the determination of node membership values is performed using the union operation based on the rule from (Sijabat et al., 2025) as follows.

$$\sigma_{\tilde{R} \cup \tilde{S} \cup \tilde{T}} = \text{maks} \{ \sigma_{\tilde{R}}(x), \sigma_{\tilde{S}}(x), \sigma_{\tilde{T}}(x) \}$$

Based on the above formulation, the membership value of node A during

the morning period for the value $x = 89$ is as follows.

$$\begin{aligned} & \sigma_{\text{Low} \cup \text{Medium} \cup \text{High}}(89) \\ &= \text{maks} \{ \sigma_{\text{Low}}(89), \sigma_{\text{Medium}}(89), \sigma_{\text{High}}(89) \} \\ &= \sigma_{\text{Low}}(89) \end{aligned}$$

Thus, node A has a linguistic membership value of Low, and through the same process, the membership values for the other nodes in the fuzzy graph $G(V,E)$ are obtained, as shown in the following table.

The membership values of the edges in the fuzzy graph are determined based on the membership values of the nodes. According to Sijabat et al. (2025), these values are divided into three categories, similar to the nodes, namely Low (L), Medium (M), and High (H), as formulated in the following equation.

$$\mu(v_i, v_j) = \begin{cases} \tilde{L}, & \text{if } (\sigma(v_i) = \sigma(v_j) = \tilde{R}) \text{ or } (\sigma(v_i) = \tilde{L} \wedge \sigma(v_j) = \tilde{M}) \text{ or } (\sigma(v_i) = \tilde{M} \wedge \sigma(v_j) = \tilde{L}) \text{ or } (\sigma(v_i) = \tilde{H} \wedge \sigma(v_j) = \tilde{L}) \\ \tilde{M}, & \text{if } (\sigma(v_i) = \tilde{M} \wedge \sigma(v_j) = \tilde{H}) \text{ or } (\sigma(v_i) = \tilde{H} \wedge \sigma(v_j) = \tilde{M}) \text{ or } (\sigma(v_i) = \sigma(v_j) = \tilde{M}) \\ \tilde{H}, & \text{if } \sigma(v_i) = \sigma(v_j) = \tilde{H} \end{cases}$$

Referring to the equation above, the linguistic membership degrees of the edges of the fuzzy graph in the morning can be obtained as follow.

$$\mu(v_B, v_F) = \tilde{L}, \text{ because } \sigma(v_B) = \tilde{M} \wedge \sigma(v_F) = \tilde{L}$$

Using the same method, all linguistic values in Table 8 and Table 9 are also determined.

Table 8. Fuzzy Graph Node Membership Values at 08.00 – 09.00

Intersection	Movement	q	$\sigma(x)$ Numerical	$\sigma(x)$ Linguistic
1	A	89	0.78	R
1	B	390	0.78	S
1	C	40	0.96	R

1	D	30	1.00	R
1	E	386	0.79	S
1	F	121	0.66	R
1	G	34	0.99	R
1	H	95	0.76	R
1	I	33	0.99	R
1	J	140	0.59	R
1	K	79	0.82	R
1	L	169	0.51	S
2	M	135	0.61	R
2	N	461	0.60	S
2	O	108	0.71	R
2	P	27	1.00	R
2	Q	591	0.73	T
2	R	90	0.78	R
2	S	34	0.99	R
2	T	130	0.63	R
2	U	109	0.71	R
2	V	93	0.77	R
2	W	153	0.54	R
2	X	91	0.77	R

Table 9. Fuzzy Graph Node Membership Values at 15.30 – 16.30

Intersection	Movement	q	$\sigma(x)$	$\sigma(x)$
			Numerical	Linguistic
1	A	128	0.64	R
1	B	601	0.75	T
1	C	61	0.89	R
1	D	34	0.99	R
1	E	624	0.81	T
1	F	101	0.74	R
1	G	85	0.80	R
1	H	159	0.52	R
1	I	75	0.83	R
1	J	168	0.51	S
1	K	90	0.78	R
1	L	139	0.60	R
2	M	155	0.54	R
2	N	719	0.80	T
2	O	118	0.67	R
2	P	44	0.95	R
2	Q	627	0.82	T
2	R	96	0.76	R
2	S	136	0.61	R
2	T	187	0.58	S
2	U	36	0.98	R
2	V	145	0.57	R
2	W	220	0.70	S
2	X	120	0.67	R

The optimization of traffic phase distribution on Prof. Dr. Jhon Aryo Katili Road is carried out by applying the fuzzy graph coloring technique based on the theory developed by (Sijabat et al., 2025). By determining the maximum chromatic number, this approach serves to identify the most effective phase arrangement solution to minimize vehicle conflict points in the area.

Referring to the node and edge membership values, the linguistic sets of node degree (σ) and edge degree (μ) for the morning and afternoon periods can be obtained as follows.

$$L_{\sigma} = \{R, S, T\}$$

$$L_{\mu} = \{R, S, T\}$$

With $\alpha = H$ (High), the structure of the fuzzy graph formed is $G = (V, E, \sigma, \mu)$ with the set elements as follows.

For the fuzzy graph G_T in the morning with $\alpha = T$, the set of nodes $V_T = \{Q\}$ and the set of edges $E = \emptyset$ are obtained. This indicates that only node Q has a high membership degree and has no other adjacent nodes. Therefore, node Q is assigned the color blue as the single color in the graph.



Figure 6. Coloring of the Fuzzy Graph G_T in the Morning

Based on Figure 4, which displays the coloring of the fuzzy graph G_T in the morning, it is known that the node set $V_T = \{v17\}$ produces a family of sets $\Gamma = \{\gamma17\}$ with $\gamma17 = \{v17\}$.

Given that only one color set group is formed, $\gamma17$ is directly colored blue. This node coloring indicates that the chromatic number for the graph G_T in the morning is $\chi = 1$. Meanwhile, the formation results of the fuzzy graph G_T in the afternoon with $\alpha = T$ are as follows:

$$V_T = \{v_i | \sigma_L(v_i) = T\} = \{B, E, N, Q\}$$

$$E_T = \{(x, y) | \mu_L(x, y) = T\} = \emptyset$$

In the principle of graph coloring, nodes B, E, N, and Q are given the same color because there is no adjacency relationship or no connecting edges between these nodes. Thus, the coloring results are shown as follows.



Figure 7. Coloring of the Fuzzy Graph G_T in the Afternoon

Figure 7 shows the results of fuzzy graph coloring on the graph G_T in the afternoon, with $V_T = \{v_2, v_5, v_{14}, v_{17}\}$. From these nodes, a family of sets $\Gamma = \{\gamma_2, \gamma_5, \gamma_{14}, \gamma_{17}\}$ is formed, where

$$\gamma_2 = \{v_2, v_5, v_{14}, v_{17}\}$$

$$\gamma_5 = \{v_5, v_2, v_{14}, v_{17}\}$$

$$\gamma_{14} = \{v_{14}, v_2, v_5, v_{17}\}$$

$$\gamma_{17} = \{v_{17}, v_2, v_5, v_{14}\}$$

Because the values obtained for $\gamma_2, \gamma_5, \gamma_{14},$ and γ_{17} are the same, it is sufficient to take the set γ_2 to be colored blue. Given that all nodes have been successfully colored within that single group, the result obtained is that the chromatic number of the graph G_T in the afternoon is $X_T = 1$.

Using the same method as for $\alpha = T$, for $\alpha = S$ in the morning we obtain $G_S = (V_S, E_S, \sigma_S, \mu_S)$ which yields.

$$V_S = \{v_i | \sigma_S(v_i) = S\} = \{B, E, L, N\}$$

$$E_S = \{(x, y) | \mu_S(x, y) = S\} = \{BL, EL, QL, QB\}$$

There are 4 nodes that have a medium membership value, denoted as follows.

$$V = \{v_2, v_5, v_{12}, v_{14}\}$$

Then a family of sets is constructed by defining the family as follows:

$$\Gamma = \{\gamma_2, \gamma_5, \gamma_{12}, \gamma_{14}\}$$

Next, the members of each set are determined by grouping the nodes that are not adjacent to each other as follows.

$$\gamma_2 = \{v_2, v_5, v_{14}\}$$

$$\gamma_5 = \{v_5, v_{14}, v_2\}$$

$$\gamma_{12} = \{v_{12}, v_{14}\}$$

$$\gamma_{14} = \{v_{14}, v_{12}\}$$

There are several identical γ values, namely $\gamma_2 = \gamma_5$ dan $\gamma_{12} = \gamma_{14}$. Therefore, simplification is performed by taking one node to represent each identical group as follows.

$$\Gamma = \{\gamma_2, \gamma_{12}\}$$

Because the two sets γ_2 and γ_{12} have different values, they are given different colors, where γ_2 is colored red and γ_{12} colored purple, as shown in the figure below.

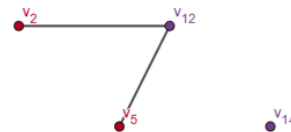


Figure 8. Coloring of the Fuzzy Graph G_S in the Morning

Because all nodes have been successfully colored, the chromatic number of the graph G_S in the morning $\chi_S = 2$. Meanwhile, the formation results of the fuzzy graph G_S in the afternoon with $\alpha = S$, are as follows.

$$V_S = \{v_i | \sigma_S(v_i) = S\} = \{J, T, W\}$$

$$E_S = \{(x, y) | \mu_S(x, y) = S\} = \{BL, BJ, EL, NT, NW, QT, QW, QL, QB\}$$

There are 3 nodes that have a medium membership value, denoted as follows.

$$V = \{v_{10}, v_{20}, v_{23}\}$$

Then a family of sets is constructed by defining the family as follows.

$$\Gamma = \{\gamma_{10}, \gamma_{20}, \gamma_{23}\}$$

Next, the members of each set are determined by grouping the nodes that are not adjacent to each other as follows.

$$\gamma_{10} = \{v_{10}, v_{20}, v_{23}\}$$

$$\gamma_{20} = \{v_{20}, v_{23}, v_{10}\}$$

$$\gamma_{23} = \{v_{23}, v_{20}, v_{10}\}$$

There are identical γ values, namely $\gamma_{10} = \gamma_{20} = \gamma_{23}$. Therefore, one node is taken to represent the same group as follows.

$$\Gamma = \{\gamma_{10}\}$$

Because these three nodes have the same value, they can be represented by a single color. Therefore, the representation and the coloring results of the fuzzy graph G_S for $\alpha = S$ can be seen in the following figure.



Figure 9. Coloring of the Fuzzy Graph G_S in the Afternoon

After all nodes have been properly accommodated into a single coloring, the chromatic number for the graph G_S in the afternoon is $\chi_S = 1$.

Then for $\alpha = R$ in the morning, we obtain $G_R = (V_R, E_R, \sigma_R, \mu_R)$ along with its set elements:

$$V_R = \{v_i | \sigma_L(v_i)\} \\ = \{A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X\}$$

$$E_R = \{(x, y) | \mu_L(x, y) \geq R\} \\ = \left\{ \begin{array}{l} AF, AQ, AH, BF, BH, BI, BK, CE, CH, \\ BT, BW, BV, BJ, BR, BU, BP, CD, GK, \\ CI, CK, CL, DK, EG, EH, EI, EK \\ FI, FH, FK, FL, HL, IK, LI, MT \\ IT, IW, IX, IV, IR, IQ, IU, IJ, LG \\ JT, JW, JX, JV, JR, JQ, JU, JP, MR \\ NW, NR, NT, NU, NX, NV, OQ, OT \\ OU, OW, OX, OP, PW, QS, QU \\ QK, QC, QH, QG, QT, QW, QX, SX \\ SK, SL, SC, SI, SH, SG, SB, \\ RT, RU, RW, RX, TX, UW, UV, XU \\ XK, XL, XC, XI, XH, XG, XB, \end{array} \right\}$$

There are 24 nodes that make up the fuzzy graph on Jhon Aryo Katili Road. All of these nodes have now been labeled with the latest notation as follows.

$$V = \{v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 v_9 v_{10} v_{11} v_{12} v_{13} v_{14} \} \\ \{v_{15} v_{16} v_{17} v_{18} v_{19} v_{20} v_{21} v_{22} v_{23} v_{24} \}$$

Then a family of sets is constructed by defining the family Γ as follows:

$$\Gamma = \left\{ \begin{array}{l} \gamma_1 \gamma_2 \gamma_3 \gamma_4 \gamma_5 \gamma_6 \gamma_7 \gamma_8 \gamma_9 \gamma_{10} \gamma_{11} \gamma_{12} \gamma_{13} \gamma_{14} \\ \gamma_{15} \gamma_{16} \gamma_{17} \gamma_{18} \gamma_{19} \gamma_{20} \gamma_{21} \gamma_{22} \gamma_{23} \gamma_{24} \end{array} \right\}$$

Next, the members of each set γ are determined by grouping the nodes that are not adjacent to each other as follows.

$$\gamma_1 = \{v_1, v_2, v_3, v_{13}, v_{14}, v_{15}\}$$

$$\gamma_2 = \{v_2, v_3, v_{13}, v_{14}, v_{15}, v_7\}$$

$$\gamma_3 = \{v_3, v_{13}, v_{14}, v_{15}, v_7, v_2\}$$

$$\gamma_4 = \{v_4, v_5, v_6, v_{10}, v_{13}, v_{14}\}$$

$$\gamma_5 = \{v_5, v_6, v_{10}, v_{13}, v_{14}, v_{15}\}$$

$$\gamma_6 = \{v_6, v_{10}, v_{13}, v_{14}, v_{15}, v_5\}$$

$$\gamma_7 = \{v_7, v_8, v_9, v_{13}, v_{14}, v_{15}\}$$

$$\gamma_8 = \{v_8, v_9, v_{13}, v_{14}, v_{15}, v_4\}$$

$$\gamma_9 = \{v_9, v_{13}, v_{14}, v_{15}, v_4, v_8\}$$

$$\gamma_{10} = \{v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}\}$$

$$\gamma_{11} = \{v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_1\}$$

$$\gamma_{12} = \{v_{12}, v_{13}, v_{14}, v_{15}, v_1, v_{11}\}$$

$$\gamma_{13} = \{v_{13}, v_{14}, v_{15}, v_1, v_4, v_5\}$$

$$\gamma_{14} = \{v_{14}, v_{15}, v_1, v_4, v_5, v_6\}$$

$$\gamma_{15} = \{v_{15}, v_1, v_4, v_5, v_6, v_{14}\}$$

$$\gamma_{16} = \{v_{16}, v_{17}, v_{18}, v_{22}, v_4, v_5\}$$

$$\gamma_{17} = \{v_{17}, v_{18}, v_{22}, v_4, v_5, v_6\}$$

$$\gamma_{18} = \{v_{18}, v_{22}, v_4, v_5, v_6, v_{17}\}$$

$$\gamma_{19} = \{v_{19}, v_{20}, v_{21}, v_4, v_5, v_6\}$$

$$\gamma_{20} = \{v_{20}, v_{21}, v_4, v_5, v_6, v_{16}\}$$

$$\gamma_{21} = \{v_{21}, v_4, v_5, v_6, v_{16}, v_{20}\}$$

$$\gamma_{22} = \{v_{22}, v_{23}, v_{24}, v_4, v_5, v_6\}$$

$$\gamma_{23} = \{v_{23}, v_{24}, v_4, v_5, v_6, v_{13}\}$$

$$\gamma_{24} = \{v_{24}, v_4, v_5, v_6, v_{13}, v_{23}\}$$

From this set formation process, identical members are obtained, namely,

$$\gamma_2 = \gamma_3, \gamma_5 = \gamma_6, \gamma_8 = \gamma_9, \gamma_{11} = \gamma_{12}, \gamma_{14} = \gamma_{15}, \gamma_{17} = \gamma_{18}, \gamma_{20} = \gamma_{21}, \gamma_{23} = \gamma_{24}.$$

Because there are identical members among several γ , simplification is performed by selecting one representative from each identical group as follows.

$$\Gamma = \{\gamma_1, \gamma_2, \gamma_4, \gamma_5, \gamma_7, \gamma_8, \gamma_{10}, \gamma_{11}, \gamma_{13}, \gamma_{14}, \gamma_{16}, \gamma_{17}, \gamma_{19}, \gamma_{20}, \gamma_{22}, \gamma_{23}\}$$

The main requirement for forming the family of sets Γ is that every pair of sets within it must satisfy the condition $\gamma_i \cap \gamma_j = \emptyset; i \neq j; i, j = 1, 2, \dots, \kappa$, yaitu $\gamma_{17} \cap \gamma_1, \gamma_{17} \cap \gamma_8, \gamma_{17} \cap \gamma_9, \gamma_{17} \cap \gamma_{10}, \gamma_{17} \cap \gamma_{11}, \gamma_{17} \cap \gamma_{12}, \gamma_{18} \cap \gamma_1, \gamma_{18} \cap \gamma_8, \gamma_{18} \cap \gamma_9, \gamma_{18} \cap \gamma_{10}, \gamma_{18} \cap \gamma_{11}, \gamma_{18} \cap \gamma_{12}, \gamma_{19} \cap \gamma_5, \gamma_{19} \cap \gamma_6, \gamma_{19} \cap \gamma_7, \gamma_{19} \cap \gamma_8, \gamma_{19} \cap \gamma_9, \gamma_{19} \cap \gamma_{10}, \gamma_{19} \cap \gamma_{11}, \gamma_{20} \cap \gamma_6, \gamma_{20} \cap \gamma_7, \gamma_{20} \cap \gamma_8, \gamma_{20} \cap \gamma_9, \gamma_{20} \cap \gamma_{10}, \gamma_{20} \cap \gamma_{11}, \gamma_{20} \cap \gamma_{12}, \gamma_{21} \cap \gamma_7, \gamma_{21} \cap \gamma_8, \gamma_{21} \cap \gamma_9, \gamma_{21} \cap \gamma_{10}, \gamma_{21} \cap \gamma_{11}, \gamma_{21} \cap \gamma_{12}, \gamma_{22} \cap \gamma_7, \gamma_{22} \cap \gamma_8, \gamma_{22} \cap \gamma_9, \gamma_{22} \cap \gamma_{10}, \gamma_{22} \cap \gamma_{11}, \gamma_{22} \cap \gamma_{12}$.

Thus, 35 pairs were found that satisfy the requirement and have 6 members of the node intersection that are maximum in value. Therefore, from the set Γ , a pair can be selected, namely $\gamma_1 \cap \gamma_{16}$ to undergo the coloring process. This yields the family of sets $\Gamma' = \gamma_1 \cap \gamma_{16}$, where γ_1 is colored red and γ_{16} is colored green. The coloring of the remaining nodes is carried out as follows.

$$S = V - \bigcup_{j=1}^k \gamma_j$$

$$= \{v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 v_9 v_{10} v_{11} v_{12} v_{13} v_{14}\}$$

$$= \{v_{15} v_{16} v_{17} v_{18} v_{19} v_{20} v_{21} v_{22} v_{23} v_{24}\}$$

$$- \{v_1, v_2, v_3, v_{13}, v_{14}, v_{15}, v_{16}, v_{17}, v_{18}, v_{22}, v_4, v_5\}$$

$$= \{v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{19}, v_{20}, v_{21}, v_{23}, v_{24}\}$$

Based on the above results, 12 nodes remained uncolored. Therefore, a new family of sets is formed, namely

$$\Gamma' = \{\gamma_6^*, \gamma_7^*, \gamma_8^*, \gamma_9^*, \gamma_{10}^*, \gamma_{11}^*, \gamma_{12}^*, \gamma_{19}^*, \gamma_{20}^*, \gamma_{21}^*, \gamma_{23}^*, \gamma_{24}^*\}$$

Where the members of the set are nodes that are not adjacent to each other within a set, following the same steps as before. Thus, all nodes in the graph were successfully colored.

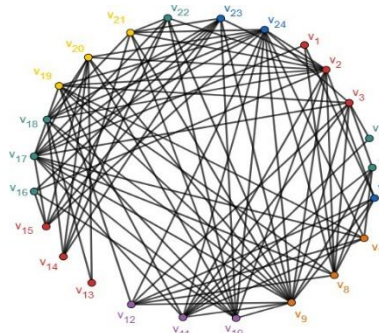


Figure 10. Coloring of the Fuzzy Graph G_R on Prof. Dr. Jhon Aryo Katili Road in the Morning

Because all nodes have been successfully colored, the chromatic number of the graph G_R in the morning is $X_R = 8$.

$$\gamma_1(i) = \begin{cases} S, & \text{if } i = B, \\ R, & \text{if } i = A, C \end{cases}$$

$$\gamma_4(i) = \begin{cases} S, & \text{if } i = E, \\ R, & \text{if } i = D, F \end{cases}$$

$$\gamma_7(i) = R, \quad \text{if } i = G, H, I$$

$$\gamma_{10}(i) = \begin{cases} S, & \text{if } i = L, \\ R, & \text{if } i = J, K \end{cases}$$

$$\gamma_{13}(i) = \begin{cases} S, & \text{if } i = N, \\ R, & \text{if } i = M, O \end{cases}$$

$$\gamma_{16}(i) = \begin{cases} T, & \text{if } i = Q, \\ R, & \text{if } i = P, R \end{cases}$$

$$\gamma_{19}(i) = R, \quad \text{if } i = S, T, U$$

$$\gamma_{22}(i) = R, \quad \text{if } i = V, W, X$$

Next, for $\alpha = R$ in the afternoon, we obtain $GR = (VR, ER, \sigma R, \mu R)$ along with its set elements.

$$V_R = \{v_i | \sigma_L(v_i)\} \\ = \{A, B, C, D, E, F, G, H, I, J, K, L, \\ M, N, O, P, Q, R, S, T, U, V, W, X\}$$

$$E_R = \{(x, y) | \mu_L(x, y) \geq R\} \\ = \left\{ \begin{array}{l} AF, AQ, AH, BF, BH, BI, BK, CE, CH, \\ BT, BW, BV, BR, BU, BP, CD, GK, \\ CI, CK, CL, DK, EG, EH, EI, EK, \\ FI, FH, FK, FL, HL, IK, LI, MT \\ IT, IW, IX, IV, IR, IQ, IU, IJ, LG \\ JT, JW, JX, JV, JR, JQ, JU, JP, MR \\ NW, NR, NU, NX, NV, OQ, OT \\ OU, OW, OX, OP, PW, QS, QU \\ QK, QC, QH, QG, QX, SX \\ SK, SL, SC, SI, SH, SG, SB, \\ RT, RU, RW, RX, TX, UW, UV, XU \\ XK, XL, XC, XI, XH, XG, XB, \end{array} \right\}$$

There are 24 nodes that make up the fuzzy graph on Jhon Aryo Katili Road. All of these nodes have now been labeled with the latest notation as follows.

$$V = \{v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 v_9 v_{10} v_{11} v_{12} v_{13} v_{14} \\ v_{15} v_{16} v_{17} v_{18} v_{19} v_{20} v_{21} v_{22} v_{23} v_{24}\}$$

Then a family of sets is constructed by defining the family Γ as follows.

$$\Gamma = \left\{ \begin{array}{l} \gamma_1 \gamma_2 \gamma_3 \gamma_4 \gamma_5 \gamma_6 \gamma_7 \gamma_8 \gamma_9 \gamma_{10} \gamma_{11} \gamma_{12} \gamma_{13} \gamma_{14} \\ \gamma_{15} \gamma_{16} \gamma_{17} \gamma_{18} \gamma_{19} \gamma_{20} \gamma_{21} \gamma_{22} \gamma_{23} \gamma_{24} \end{array} \right\}$$

Next, the members of each set γ are determined by grouping the nodes that are not adjacent to each other as follows.

$$\begin{aligned} \gamma_1 &= \{v_1, v_2, v_3, v_{13}, v_{14}, v_{15}\} \\ \gamma_2 &= \{v_2, v_3, v_{13}, v_{14}, v_{15}, v_7\} \\ \gamma_3 &= \{v_3, v_{13}, v_{14}, v_{15}, v_7, v_2\} \\ \gamma_4 &= \{v_4, v_5, v_6, v_{10}, v_{13}, v_{14}\} \\ \gamma_5 &= \{v_5, v_6, v_{10}, v_{13}, v_{14}, v_{15}\} \\ \gamma_6 &= \{v_6, v_{10}, v_{13}, v_{14}, v_{15}, v_5\} \\ \gamma_7 &= \{v_7, v_8, v_9, v_{13}, v_{14}, v_{15}\} \\ \gamma_8 &= \{v_8, v_9, v_{13}, v_{14}, v_{15}, v_4\} \\ \gamma_9 &= \{v_9, v_{13}, v_{14}, v_{15}, v_4, v_8\} \\ \gamma_{10} &= \{v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}\} \\ \gamma_{11} &= \{v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_1\} \\ \gamma_{12} &= \{v_{12}, v_{13}, v_{14}, v_{15}, v_1, v_{11}\} \\ \gamma_{13} &= \{v_{13}, v_{14}, v_{15}, v_1, v_4, v_5\} \\ \gamma_{14} &= \{v_{14}, v_{15}, v_1, v_4, v_5, v_6\} \\ \gamma_{15} &= \{v_{15}, v_1, v_4, v_5, v_6, v_{14}\} \\ \gamma_{16} &= \{v_{16}, v_{17}, v_{18}, v_{22}, v_4, v_5\} \\ \gamma_{17} &= \{v_{17}, v_{18}, v_{22}, v_4, v_5, v_6\} \\ \gamma_{18} &= \{v_{18}, v_{22}, v_4, v_5, v_6, v_{17}\} \end{aligned}$$

$$\begin{aligned} \gamma_{19} &= \{v_{19}, v_{20}, v_{21}, v_4, v_5, v_6\} \\ \gamma_{20} &= \{v_{20}, v_{21}, v_4, v_5, v_6, v_{16}\} \\ \gamma_{21} &= \{v_{21}, v_4, v_5, v_6, v_{16}, v_{20}\} \\ \gamma_{22} &= \{v_{22}, v_{23}, v_{24}, v_4, v_5, v_6\} \\ \gamma_{23} &= \{v_{23}, v_{24}, v_4, v_5, v_6, v_{13}\} \\ \gamma_{24} &= \{v_{24}, v_4, v_5, v_6, v_{13}, v_{23}\} \end{aligned}$$

From this set formation process, identical members were obtained, namely $\gamma_2 = \gamma_3$, $\gamma_5 = \gamma_6$, $\gamma_8 = \gamma_9$, $\gamma_{11} = \gamma_{12}$, $\gamma_{14} = \gamma_{15}$, $\gamma_{17} = \gamma_{18}$, $\gamma_{20} = \gamma_{21}$, $\gamma_{23} = \gamma_{24}$. Because there are identical members among several γ , simplification is performed by selecting one representative from each identical group as follows.

$$\Gamma = \left\{ \begin{array}{l} \gamma_1, \gamma_2, \gamma_4, \gamma_5, \gamma_7, \gamma_8, \gamma_{10}, \gamma_{11}, \gamma_{13}, \gamma_{14}, \\ \gamma_{16}, \gamma_{17}, \gamma_{19}, \gamma_{20}, \gamma_{22}, \gamma_{23} \end{array} \right\}$$

The main requirement for forming the family of sets Γ is that every pair of sets within it must satisfy the condition $\gamma_i \cap \gamma_j = \emptyset$; $i \neq j$; $i, j = 1, 2, \dots, \kappa$, namely $\gamma_{17} \cap \gamma_1, \gamma_{17} \cap \gamma_8, \gamma_{17} \cap \gamma_9, \gamma_{17} \cap \gamma_{10}, \gamma_{17} \cap \gamma_{11}, \gamma_{17} \cap \gamma_{12}, \gamma_{18} \cap \gamma_1, \gamma_{18} \cap \gamma_8, \gamma_{18} \cap \gamma_9, \gamma_{18} \cap \gamma_{10}, \gamma_{18} \cap \gamma_{11}, \gamma_{18} \cap \gamma_{12}, \gamma_{19} \cap \gamma_5, \gamma_{19} \cap \gamma_6, \gamma_{19} \cap \gamma_7, \gamma_{19} \cap \gamma_8, \gamma_{19} \cap \gamma_9, \gamma_{19} \cap \gamma_{10}, \gamma_{19} \cap \gamma_{11}, \gamma_{20} \cap \gamma_6, \gamma_{20} \cap \gamma_7, \gamma_{20} \cap \gamma_8, \gamma_{20} \cap \gamma_9, \gamma_{20} \cap \gamma_{10}, \gamma_{20} \cap \gamma_{11}, \gamma_{20} \cap \gamma_{12}, \gamma_{21} \cap \gamma_7, \gamma_{21} \cap \gamma_8, \gamma_{21} \cap \gamma_9, \gamma_{21} \cap \gamma_{10}, \gamma_{21} \cap \gamma_{11}, \gamma_{21} \cap \gamma_{12}, \gamma_{22} \cap \gamma_7, \gamma_{22} \cap \gamma_8, \gamma_{22} \cap \gamma_9, \gamma_{22} \cap \gamma_{10}, \gamma_{22} \cap \gamma_{11}, \gamma_{22} \cap \gamma_{12}$.

Thus, 35 pairs were found that satisfy the requirement and have 6 members of the node intersection that are maximum in value. Therefore, from the set Γ , a pair can be selected, namely $\gamma_1 \cap \gamma_{16}$ to undergo the coloring process. This yields the family of sets $\Gamma' = \gamma_1 \cap \gamma_{16}$, where γ_1 is colored yellow and γ_{16} is colored purple. The coloring of the remaining nodes is carried out as follows.

$$S = V - \bigcup_{j=1}^k \gamma_j \\ = \{v_1 v_2 v_3 v_4 v_5 v_6 v_7 v_8 v_9 v_{10} v_{11} v_{12} v_{13} v_{14} \\ v_{15} v_{16} v_{17} v_{18} v_{19} v_{20} v_{21} v_{22} v_{23} v_{24}\}$$

$$-\{v_1, v_2, v_3, v_{13}, v_{14}, v_{15}, v_{16}, v_{17}, v_{18}, v_{22}, v_4, v_5\} \\ = \{v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{19}, v_{20}, v_{21}, v_{23}, v_{24}\}$$

Based on the above results, 12 nodes remained uncolored. Therefore, a new family of sets is formed, namely:

$$\Gamma' = \{\gamma_6^*, \gamma_7^*, \gamma_8^*, \gamma_9^*, \gamma_{10}^*, \gamma_{11}^*, \gamma_{12}^*, \gamma_{19}^*, \gamma_{20}^*, \gamma_{21}^*, \gamma_{23}^*, \gamma_{24}^*\}$$

Where the members of the set are nodes that are not adjacent to each other within a set, following the same steps as before. Thus, all nodes in the graph were successfully colored.

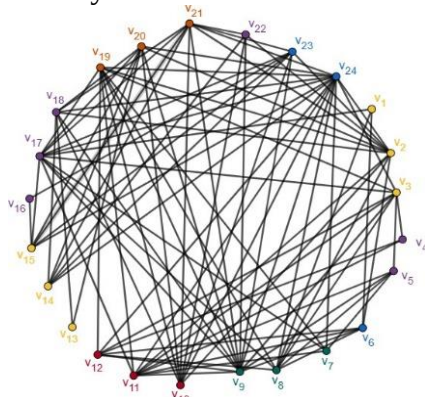


Figure 11. Coloring of the Fuzzy Graph G_R on Prof. Dr. Jhon Aryo Katili Road in the Afternoon

Because all nodes have been successfully colored, the chromatic number of the graph G_R in the morning is $X_R = 8$.

$$\begin{aligned} \gamma_1(i) &= \begin{cases} S, & \text{if } i = B, \\ R, & \text{if } i = A, C \end{cases} \\ \gamma_4(i) &= \begin{cases} S, & \text{if } i = E, \\ R, & \text{if } i = D, F \end{cases} \\ \gamma_7(i) &= R, & \text{if } i = G, H, I \\ \gamma_{10}(i) &= \begin{cases} S, & \text{if } i = L, \\ R, & \text{if } i = J, K \end{cases} \\ \gamma_{13}(i) &= \begin{cases} S, & \text{if } i = N, \\ R, & \text{if } i = M, O \end{cases} \\ \gamma_{16}(i) &= \begin{cases} T, & \text{if } i = Q, \\ R, & \text{if } i = P, R \end{cases} \\ \gamma_{19}(i) &= R, & \text{if } i = S, T, U \\ \gamma_{22}(i) &= R, & \text{if } i = V, W, X \end{aligned}$$

Based on the evaluation of the fuzzy graph coloring, a solution set is

obtained that forms a chromatic number $\chi(G) = 8$. This represents the existence of eight traffic regulation phases at the Bundaran Digulis intersection, the details of which can be seen in the following table.

Table 10. Traffic Phases on Prof. Dr. Jhon Aryo Katili Road

Fase 1	Fase 2	Fase 3	Fase 4
v_1, v_2, v_3 atau A, B, C	v_4, v_5, v_6 atau D, E, F	v_7, v_8, v_9 atau G, H, I	v_{10}, v_{11}, v_{12} atau J, K, L
Fase 5	Fase 6	Fase 7	Fase 8
v_{13}, v_{14}, v_{15} atau M, N, O	v_{16}, v_{17}, v_{18} atau P, Q, R	v_{19}, v_{20}, v_{21} atau S, T, U	v_{22}, v_{23}, v_{24} atau V, W, X

The traffic conditions at the Bundaran Digulis intersection in Pontianak can be modeled through graph coloring, where each color represents a specific flow phase. Based on Figure 8, there are eight movement phases regulated using this approach.

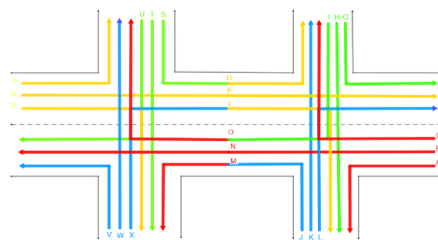


Figure 12. Traffic Route Mapping Using Fuzzy Graph Coloring on Prof. Dr. Jhon Aryo Katili Road

Figure 10 shows the representation of traffic flow at two intersections (Intersection 1 and Intersection 2) with four main approaches, namely east, west, north, and south, where each flow coded with letters (A–X) describes vehicle movement from one road to another according to the table. At Intersection 1, flows come from Prof. Dr. Jhon Aryo Katili Road, Yusuf Hasiru Road, and Arif Rahman Hakim Road, which are interconnected through straight or

turning movements, while at Intersection 2, flows connect Prof. Dr. Jhon Aryo Katili Road, Tondano Road, and Madura Road with similar movement patterns. Overall, this figure systematically illustrates the origin–destination relationships of vehicles as a basis for analysis and traffic regulation at both intersections.

CONCLUSION

This study shows that the application of fuzzy graphs in traffic light regulation can optimize the distribution of phases more adaptively based on traffic density levels, thereby reducing conflicts and improving traffic flow compared to the previous system. For future research, it is recommended to add variables such as waiting time and daily traffic patterns and integrate them with an intelligent system, as well as to test the model on more diverse locations so that the results are more broadly applicable and practical.

REFERENCES

- Arifai, A. A., & Juniati, D. (2013). *Hutan dan sikel pada graf fuzzy. MATHunesa: Jurnal Ilmiah Matematika*, 1(5).
- Fanani, A. (2016). Optimasi waktu tunggu lampu lalu lintas dengan menggunakan graf kompatibel sebagai upaya mengurangi kemacetan. *SYSTEMIC: Information System and Informatics Journal*, 2(1), 45–50.
- Fuad, A., & Rahadjeng, B. (2014). Pewarnaan titik pada graf fuzzy. *MATHunesa: Jurnal Ilmiah Matematika*, 3(1).
- Kaviya, V., & Ramya, M. (2022). *Spherical fuzzy graph application in traffic. International Journal for Research in Applied Science & Engineering Technology (IJRASET)*, 10(11).

<https://doi.org/10.22214/ijraset.2022.47626>

- Kurniawan, A. P. (2017). Aplikasi Graf Fuzzy Dan Aljabar Max-Plus Untuk Pengaturan Fuzzy Graph and Max-Plus Algebra Application for Traffic Light in Beran. *Jurnal Matematika*, 6(2), 72–86.
- Manual Kapasitas Jalan Indonesia. (1997). *Manual Kapasitas Jalan Indonesia (MKJI)*. Jakarta, Indonesia: Direktorat Bina Jalan Kota (Binkot), Proyek Highway Capacity Manual Phase 3.
- Pobela, D. S., Rachman, A., & Ursilu, S. (2020). Analisis kemacetan lalu lintas di ruas Jalan Prof. Dr. H. Aloe Saboe Kota Gorontalo (Studi kasus ruas Jalan Pasar Moodu). *RADIAL: Jurnal Peradaban Sains, Rekayasa dan Teknologi*, 8(2), 101–110.
- Rahayuningsih, S. (2018). *Teori graph dan penerapannya*. Malang: Universitas Wisnuwardhana Press.
- Ramadhani, I. N., & Munandar, A. (2024). Sturuktur Graf Fuzzy dan Aplikasinya pada Pengambilan Keputusan dalam Identifikasi Layanan Perjalanan. 13(1), 20–29. <https://doi.org/10.14421/fourier.2024.131.20-29>
- Setiawan, B., & Juniati, D. (2013). *Operasi pada graf fuzzy. MATHunesa: Jurnal Ilmiah Matematika*, 1(5).
- Sijabat, H. E., Nilamsari Kusumastuti, & Fransiskus Fran. (2025). Optimalisasi Fase Dan Durasi Lampu Lalu Lintas Bundaran Digulis Pontianak. *Jurnal Derivat: Jurnal Matematika Dan Pendidikan Matematika*, 12(1), 82–96. <https://doi.org/10.31316/j.derivat.v12i1.6551>
- Sudin, S. (2024). Fuzzy graphs: An overview. *International Journal for Multidisciplinary Research (IJFMR)*, 6(4).
- Sugarda, A. (2025). Optimasi Waktu

Tunggu Total Dan Pemodelan Arus
Lalu Lintas Dengan Metode Graf
Fuzzy Pada Persimpangan Fly Over
Jamin Ginting Medan. 1287–1293.

Wahyu, A., Cahyani, R., & Rosha, M.
(2023). Optimasi Lampu Lalu Lintas
Simpang Tabuik Kota Pariaman
Menggunakan Graf Fuzzy Berbasis
FIS Tipe Mamdani. 8(3), 16–25.

Yulipriyono, E. E. P. F., & Purwanto, D.
(2017). Perubahan nilai ekivalensi
mobil penumpang akibat perubahan
karakteristik operasional kendaraan
di Jalan Kota Semarang. *Media
Komunikasi Teknik Sipil*, 23(1), 69–
76.